



Technical Report

 ing. Andrea Starnini	Subject	Report N°	Rev.	Date	Sheet
	Pressure distribution on hole	01-20	01	10/03/2024	1 of 3
		13/04/2020	Compiled by Andrea Starnini		

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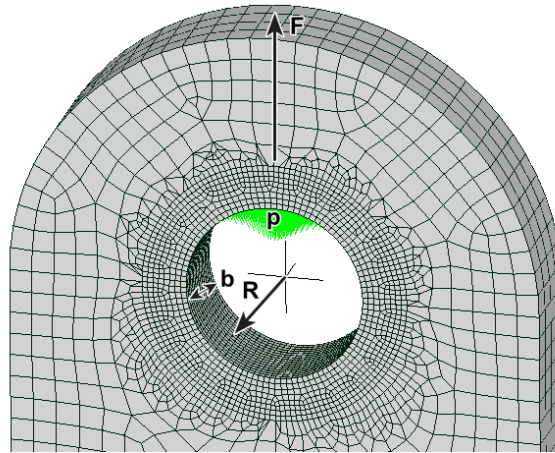
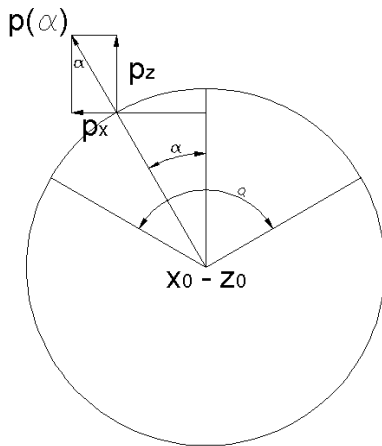
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1 Parabolic distribution

Pressure distribution $p(\alpha) = p_{\max} \cos^2\left(\frac{\pi}{a}\alpha\right)$

Vertical component $p_z(\alpha) = p_{\max} \cos^2\left(\frac{\pi}{a}\alpha\right) \cos(\alpha)$

Horizontal component $p_x(\alpha) = p_{\max} \cos^2\left(\frac{\pi}{a}\alpha\right) \sin(\alpha)$



- R Hole radius (D- diameter)
- b Plate thickness
- a Contact arc opening (° or rad)
- F Total vertical force

Hypotheses:

- Vertical force (along z)
- Constant distribution on axial direction

$$dF_z(\alpha) = p_{\max} b R d\alpha \cos^2\left(\frac{\pi}{a}\alpha\right) \cos(\alpha) \quad F = p_{\max} b R \int_{-a/2}^{a/2} \cos^2\left(\frac{\pi}{a}\alpha\right) \cos(\alpha) d\alpha$$

Integral solution: $\int_{-a/2}^{a/2} \cos^2\left(\frac{\pi}{a}\alpha\right) \cos(\alpha) d\alpha = \frac{4\pi^2 \times \text{sen}(a/2)}{4\pi^2 - a^2}$

Determination of p_{\max} $p_{\max} = \frac{F}{bR} \frac{4\pi^2 - a^2}{4\pi^2 \times \text{sen}(a/2)} = \frac{2F}{bD} \frac{4\pi^2 - a^2}{4\pi^2 \times \text{sen}(a/2)}$


Cartesian coordinates:

$$\cos^2\left(\frac{\pi}{a}\alpha\right) = \cos^2\left[\frac{\pi}{a} \times \arccos\left(\frac{z-z_0}{R}\right)\right]; \quad \cos(\alpha) = \left(\frac{z-z_0}{R}\right); \quad \sin(\alpha) = \left(\frac{x-x_0}{R}\right);$$

Vertical component $p_z(\alpha) = p_{\max} \cos^2\left[\frac{\pi}{a} \times \arccos\left(\frac{z-z_0}{R}\right)\right] \left(\frac{z-z_0}{R}\right)$

Horizontal component $p_x(\alpha) = p_{\max} \cos^2\left[\frac{\pi}{a} \times \arccos\left(\frac{z-z_0}{R}\right)\right] \left(\frac{x-x_0}{R}\right)$

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2 Co-sinusoidal distribution

Pressure distribution $p(\alpha) = p_{\max} \cos\left(\frac{\pi}{a}\alpha\right)$

Vertical component $p_z(\alpha) = p_{\max} \cos\left(\frac{\pi}{a}\alpha\right) \cos(\alpha)$

Horizontal component $p_x(\alpha) = p_{\max} \cos\left(\frac{\pi}{a}\alpha\right) \sin(\alpha)$

$dF_z(\alpha) = p_{\max} bR d\alpha \cos\left(\frac{\pi}{a}\alpha\right) \cos(\alpha)$ $F = p_{\max} bR \int_{-a/2}^{a/2} \cos\left(\frac{\pi}{a}\alpha\right) \cos(\alpha) d\alpha$

Integral solution: $\int_{-a/2}^{a/2} \cos\left(\frac{\pi}{a}\alpha\right) \cos(\alpha) d\alpha = \frac{2\pi a \cos(a/2)}{\pi^2 - a^2}$ $a \neq 180^\circ$
 $a \neq \pi$

Determination of p_{\max} $p_{\max} = \frac{F}{bR} \frac{\pi^2 - a^2}{2\pi a \cos(a/2)} = \frac{2F}{bD} \frac{\pi^2 - a^2}{2\pi a \cos(a/2)}$ $a \neq 180^\circ$
 $a \neq \pi$

$\cos\left(\frac{\pi}{a}\alpha\right) = \cos\left[\frac{\pi}{a} \times \arccos\left(\frac{z-z_0}{R}\right)\right]$; $\cos(\alpha) = \left(\frac{z-z_0}{R}\right)$; $\sin(\alpha) = \left(\frac{x-x_0}{R}\right)$;

Vertical component $p_z(\alpha) = p_{\max} \cos\left[\frac{\pi}{a} \times \arccos\left(\frac{z-z_0}{R}\right)\right] \left(\frac{z-z_0}{R}\right)$

Horizontal component $p_x(\alpha) = p_{\max} \cos\left[\frac{\pi}{a} \times \arccos\left(\frac{z-z_0}{R}\right)\right] \left(\frac{x-x_0}{R}\right)$

$a \neq 180^\circ$ $a \neq \pi$

Integral solution: $\int_{-\pi/2}^{\pi/2} \cos^2(\alpha) d\alpha = \frac{\pi}{2}$ $a = 180^\circ$

Determination of p_{\max} $p_{\max} = \frac{F}{bR} \frac{2}{\pi} = \frac{4F}{\pi bD}$ $a = 180^\circ$

Vertical component $p_z(\alpha) = p_{\max} \left(\frac{z-z_0}{R}\right)^2$

Horizontal component $p_x(\alpha) = p_{\max} \left(\frac{z-z_0}{R}\right) \left(\frac{x-x_0}{R}\right)$