where the infinite series represents the infinite modes of sloshing of the fluid; ρ_l (in kg/m³) is the mass density of the liquid, γ can be derived from Equation (2.6c), J_1 is the Bessel function of the first kind of order 1 [561] and numbers λ_n are the n−roots of the first derivative of J_1. The first ten of these roots are $\lambda_1 = 1.8412$, $\lambda_2 = 5.3314$, $\lambda_3 = 8.5363$, $\lambda_4 = 11.7060$, $\lambda_5 = 14.8636$, $\lambda_6 = 18.0155$, $\lambda_7 = 21.1644$, $\lambda_8 = 24.3113$, $\lambda_9 = 27.4571$ and $\lambda_{10} = 30.6019$ [23, 221, 596, 646]. The function ψ_n reads:

$$\psi_n = \frac{2R}{(\lambda_n^2 - 1) J_1(\lambda_n) \cosh(\lambda_n \gamma)} \tag{2.28}$$

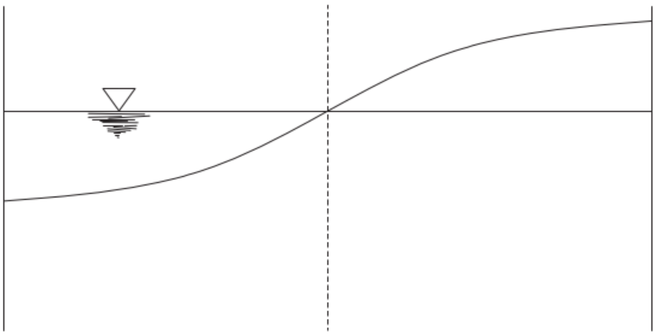
The antisymmetrical modes are depicted in Figure 2.6 by using the slosh wave shape [121, 287, 626]:

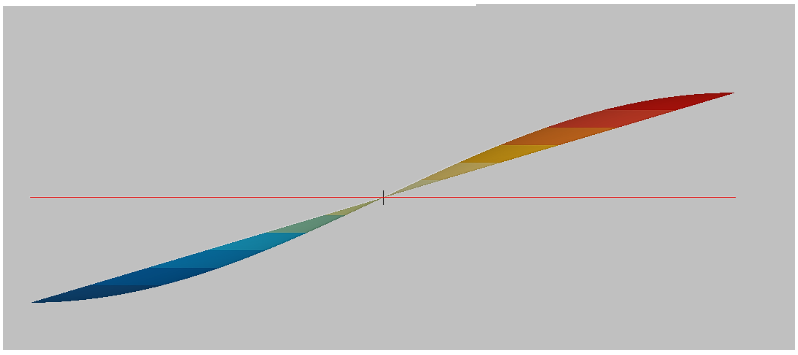
$$\frac{2RJ_1(\lambda_n \xi)}{(\lambda_n^2 - 1) J_1(\lambda_n)} \tag{2.29}$$

obtained from Equations (2.27) and (2.28) (and explained later in expression (2.85)). In the case where $n = 1$ the wave has zero amplitude at $r = 0$, a positive peak at one wall, and a negative peak at the other wall; this is usually the fundamental antisymmetric wave. The function $a_{cn}(t)$ in Equation (2.27) represents the instantaneous value of the pseudoacceleration induced by the prescribed free-field ground motion $a(t)$ in a single degree of freedom linear oscillator having the following circular frequency ω_{cn} or natural frequency f_{cn} (Section 2.2.5):

$$\omega_{cn} = \sqrt{g \frac{\lambda_n}{R} \tanh(\lambda_n \gamma)} \quad \text{or} \quad f_{cn} = \frac{1}{2\pi} \sqrt{g \frac{\lambda_n}{R} \tanh(\lambda_n \gamma)} \tag{2.30}$$

and a viscous damping ratio (Section 2.2.6), both equal to those of the n^{th}−sloshing mode of vibration of the liquid in the tank. The maximum value of the convective pressure p_c may then be determined





. The period of vibration of the first sloshing mode can be estimated using the following expression:

$$T_1 = \frac{2\pi}{\omega_{c1}} = \frac{2\pi\sqrt{\frac{R}{g}}}{\sqrt{\lambda_1 \tanh\left(\frac{\lambda_1 H}{R}\right)}} \tag{1.30}$$

The first mode sloshing wave period, in seconds, shall be calculated by Equation E.4.5.2 where K_s is the sloshing period coefficient defined in Equation E.4.5.2-c:

In SI units:

$$T_c = 1.8 K_s \sqrt{D} \tag{E.4.5.2-a}$$

or, in USC units:

$$T_c = K_s \sqrt{D} \tag{E.4.5.2-b}$$

$$K_s = \frac{0.578}{\sqrt{\tanh\left(\frac{3.68H}{D}\right)}} \tag{E.4.5.2-c}$$

The natural frequency of the convective liquid with respective wave number may be calculated as follows

$$f_{c(n,m)} = \frac{1}{2\pi}\sqrt{\lambda_{n,m} \frac{g}{R} \tanh\left(\lambda_{n,m} \frac{H_L}{R}\right)} \quad (4$$

Eurocode 8 standard introduces (4) for calculation of natural convective frequencies but th

Other sources all agree

R	10.0	[m]	Radious
D	20.0	[m]	Diameter
H	25.0	[m]	Tank High (Water Level)
ρ_p	1000	[Kg/m3]	Fluid mass density
g	9.8	[m/s2]	Gravity Constant

n	λ_n	ω_{cn}	f_{cn}	T_{cn}	Ψ_n
1	1.8412	1.34	0.21 Hz	4.68 s	0.28
2	5.3314	2.29	0.36 Hz	2.75 s	- 0.00
3	8.5363	2.89	0.46 Hz	2.17 s	0.00
4	11.7060	3.39	0.54 Hz	1.86 s	- 0.00
5	14.8636	3.82	0.61 Hz	1.65 s	0.00