Code_Aster

Version default

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Model of damage of MAZARS

Summary:

This documentation presents the model of behavior of MAZARS who allows to describe the behavior rubber band-endommageable of the concrete. This model is 3D, isotropic and is based on a criterion of damage written in deformation and describing dissymmetry traction and compression. The initial model, does not give an account of the restoration of rigidity in the event of "refermeture of the cracks" and does not take into account the possible plastic deformations or viscous effects which can be observed during deformations of a concrete. The version implemented in Code_Aster takes account of the last improvements. This reformulation of the Mazars model of the years 1980 makes it possible to better describe the behavior of the concrete in bi--compression and pure shearing.

The version $1D$ model makes it possible to give an account of the restoration of rigidity in the event of refermeture of the cracks.

Three versions of the model are established:
- the local version (with risk of dependence to the grid)
- the not-local version where the damage is controlled by the gradient of deformation. It is also possible to take into account the dependence of the parameters of the law with the temperature, the hydration and drying.
- the version $1D$ local, only used with the multifibre beams [R5.03.09].

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1 Introduction

1.1 A law of behavior élasto-endommageable

The model of behavior `MAZARS` 1] is a model simple, considered robust, based on the mechanics of the damage [feeding-bottle2] , which makes it possible to describe the reduction in the rigidity of material under the effect of the creation of microscopic cracks in the concrete. It is based on only one variable interns scalar D describing the isotropic damage of way, but distinguishing despite everything the damage from traction and the damage from compression. **The version implemented under Aster corresponds to the reformulation of 2012** [feeding-bottle3] . Major modification compared to the model of origin 1] is the introduction of a new internal variable, noted Y, corresponding to the maximum reached during the loading by the equivalent deformation defined in the years 1980. So the damage is not any more the internal variable in the revisited model. Moreover, its law of evolution is simplified in order to eliminate the concepts of damage from traction and compression.

Contrary to the model `ENDO_ISOT_BETON`, this model does not make it possible to translate the phenomenon of refermeture of the cracks (restoration of rigidity). In addition, the model of `MAZARS` does not take into account the possible plastic deformations or viscous effects which can be observed during deformations of a concrete.

The version $1D$ model of `MAZARS` is described in the document [R5.03.09] "non-linear Relations of behavior". In this specific case, the model is able to give an account of the phenomenon of refermeture of the cracks. The version $1D$ model, is usable only with the multifibre beams.

1.2 Limits of the local approach and methods of regularization

Like all the lenitive laws, the model of `MAZARS` raise difficulties related to the phenomenon of localization of the deformations.

Physically, the heterogeneity of the microstructure of the induced concrete of the remote interactions enters the formed cracks 4]. Thus, the deformations locate in a metal strip, called band of localization, to form the macro-cracks. The state of the constraints in a material point cannot be any more only described by the characteristics at the point but must also take into account its environment. In the case of the local model, no indication is included concerning the scale of cracking. Consequently, no information is not given over the bandwidth of localization which becomes worthless then. This leads to a mechanical behavior with rupture without dissipation of energy, physically unacceptable.

Mathematically, the localization returns the problem to be solved badly posed because softening causes a loss of ellipticity of the differential equations which describe the process of deformations 5]. The digital solutions do not converge towards physically acceptable solutions in spite of refinements of grid.
Numerically, one observes a dependence of the solution to the network extremely prejudicial (cf [R5.04.02]).

A method of regularization thus becomes necessary. Several are possible. The choice which was made here is to regularize in gradient of deformation, and to thus use a tensor of regularized deformation $\overline{\varepsilon}$ who checks the characteristic equation [R5.04.02]:

$$\varepsilon = \overline{\varepsilon} - L_c^2 \nabla^2 \overline{\varepsilon}$$ **(éq 1.2-1)**

where the scalar L_c (characteristic length) the dimension a length is.

Note:

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Let us announce that this model not-room does not correspond to the version initially proposed by J.Mazars and G.Pijaudier-Pooch 6] and which is in particular established in CAST3M, where delocalization is obtained while using as equivalent deformation, the average of the deformation equivalent local on a volume V :

$$\overline{\varepsilon}(x) = \frac{1}{V_r(x)} \int_{\Omega} \psi(x-s)\,\varepsilon_{eq}(s)\,ds$$

where Ω is the volume of the structure

$V_r(x)$ is representative volume at the point x : $V_r = \int_{\Omega} \psi(x-s)\,ds$

$\psi(x-s)$ is a weight function: $\psi(x-s) = \exp\left(-\dfrac{4\|x-s\|^2}{l_c^2}\right)$

l_c is an internal length (traditionally estimated at three times size of the largest aggregate).

Digital tests made it possible to connect the 2 parameters of delocalization l_c and L_c in the case of the model of Mazars. The following relation was obtained: $4L_c \simeq l_c$

The model of `MAZARS` is thus available in *Code_Aster* under 2 versions:
* the local version of the model for which the dependence of the solution to the network is observable as for all the lenitive models.
* a nonlocal version which uses a tensor of regularized deformation (known as also "nonlocal"), modeling of the type `GRAD_EPSI`.

1.3 Coupling with thermics

For certain studies, it can be interesting to be able to take into account the modification of the parameters materials under the effect of the temperature. This is possible in *Aster* (`MAZARS_FO` compound or not with `ELAS_FO`). The assumptions made for the coupling with thermics are the following ones:
* thermal dilation is supposed to be linear is:

$$\varepsilon^{th} = \alpha(T - T_{ref})\mathbf{I_d} \qquad\qquad \textbf{(éq 1.3-1)}$$

with α = constant or function of the temperature,
* one does not take into account thermomechanical interactions, i.e. one does not model the effect of the mechanical state of stress on the thermal deformation of the concrete,
* concerning the evolution of the parameters materials with the temperature, one considers that those depend not on the current temperature but on the maximum temperature $Tmax$ sight by material during its history, (effect quoted in the literature),
* only elastic strain (mechanical) induced of the damage.

Note:

Because of data-processing constraints, the initial value of $Tmax$ is initialized to 0. Consequently, one cannot use the parameters materials defined for negative temperatures (if necessary, one can however circumvent this problem while returning all the temperatures in Kelvin instead of $°C$).

1.4 Law of Mazars in the presence of a field of drying or hydration

The use of `ELAS_FO` and/or `MAZARS_FO` under the operator `DEFI_MATERIAU` allows to make depend the parameters materials on drying or hydration.

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In addition, deformations related to the withdrawal of endogenous ε_{re} and with the withdrawal desiccation ε_{rd} are taken into account in the model, in the form (linear) following (cf. [R7.01.12]) :

$$\varepsilon_{re} = -\beta\,\xi\,\mathbf{I_d} \qquad \text{(éq 1.4-1)}$$

$$\varepsilon_{rd} = -\kappa\,(C_{ref} - C)\,\mathbf{I_d} \qquad \text{(éq 1.4-2)}$$

where ξ is the hydration, C water concentration (field of drying in the terminology *Code_Aster*), C_{ref} initial water concentration (or drying of reference). Finally β is the endogenous coefficient of withdrawal and κ the coefficient of withdrawal of desiccation to be informed in `DEFI_MATERIAU`, keyword factor `ELAS_FO`, operands `B_ENDO` and `K_DESSIC`. As one said to the preceding paragraph, the choice which was made in the establishment of the model of `MAZARS`, it is that only the elastic strain induced of the damage. Consequently, if one models a concrete test-tube which dries or which is hydrated freely and uniformly, one will obtain well a field of deformation not no one and a stress field perfectly no one.

One initially presents the writing of the model then some data on the identification of the parameters. To finish, one exposes the principles of digital integration in *Code_Aster*.

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2 Models of MAZARS

2.1 Model of Origin of Mazars

The model of MAZARS was elaborate within the framework of the mechanics of the damage. This model is detailed in the thesis of MAZARS 1]
The constraint is given by the following relation:

$$\boldsymbol{\sigma} = (1 - D) \mathbf{E} \boldsymbol{\varepsilon}^{\mathbf{e}}$$ **(éq 2.1-1)**

with :

- E the matrix of Hooke,
- D the variable of damage
- . $\boldsymbol{\varepsilon}^e$ elastic strain $\boldsymbol{\varepsilon}^{\mathbf{e}} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{th} - \boldsymbol{\varepsilon}^{rd} - \boldsymbol{\varepsilon}^{re}$
- $\boldsymbol{\varepsilon}^{th} = \alpha (T - T_{ref}) \mathbf{I_d}$ thermal dilation
- $\boldsymbol{\varepsilon}^{re} = -\beta \xi \mathbf{I_d}$ endogenous withdrawal (related to the hydration)
- $\boldsymbol{\varepsilon}_{rd} = -\kappa (C_{ref} - C) \mathbf{I_d}$ withdrawal of desiccation (related to drying)

D is the variable of damage. It is understood enters 0, materials healthy, and 1, broken material. The damage is controlled by the equivalent deformation ε_{eq} who allows to translate a triaxial state by an equivalence in a uniaxial state. As the extensions are paramount in the phenomenon of cracking of the concrete, the introduced equivalent deformation is defined starting from the positive eigenvalues of the tensor of the deformations, that is to say:

$$\varepsilon_{eq} = \sqrt{\langle \boldsymbol{\varepsilon} \rangle_+ : \langle \boldsymbol{\varepsilon} \rangle_+}$$

where in the principal reference mark of the tensor of deformations: **(éq 2.1-2)**

$$\varepsilon_{eq} = \sqrt{\langle \varepsilon_1 \rangle_+^2 + \langle \varepsilon_2 \rangle_+^2 + \langle \varepsilon_3 \rangle_+^2}$$

knowing that the positive part $\langle \rangle_+$ is defined so that if ε_i is the principal deformation in the direction i :

$$\begin{cases} \langle \varepsilon_i \rangle_+ = \varepsilon_i & \text{si} \quad \varepsilon_i \geq 0 \\ \langle \varepsilon_i \rangle_+ = 0 & \text{si} \quad \varepsilon_i < 0 \end{cases}$$ **(éq 2.1-3)**

Note:

In the case of a thermomechanical loading, only elastic strain $\boldsymbol{\varepsilon}^e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{th}$ *contribute to the evolution of the damage from where:* $\varepsilon_{eq} = \sqrt{\langle \varepsilon^e \rangle_+ : \langle \varepsilon^e \rangle_+}$.

ε_{eq} is an indicator of the state of tension in the material which generates the damage. This size defines the surface of load f such as:

$$f = \varepsilon_{eq} - K(D) = 0$$ **(éq 2.1-4)**

(4)

with $K(D) = \varepsilon_{d0}$ if $D = 0$. ε_{d0} the deformation threshold of damage.
When the equivalent deformation reaches this value, the damage is activated. D is defined like a combination of two damaging modes defined by D_t and D_c, variable between 0 and 1 depending on the state of associated damage, and corresponding respectively to the damage in traction and compression. The relation binding these variables is the following one:

$$D = \alpha_t^\beta D_t + \alpha_c^\beta D_c$$ **(éq 2.1-5)**

(5)

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β is a coefficient which was introduced later on to improve behaviour in shearing. Usually its value is fixed at 1.06. Coefficients α_t and α_c carry out a link between the damage and the compactness of traction or. When traction is activated $\alpha_t = 1$ whereas $\alpha_t = 0$ and conversely in compression.

A characteristic of this model is its explicit writing what implies that all the sizes are calculated directly without using an algorithm of linearization like that of Newton-Raphson. Thus, laws of evolution of the damages D_t and D_c express themselves only starting from the equivalent deformation ε_{eq}

$$D_t = 1 - \frac{(1 - A_t)\varepsilon_{d0}}{\varepsilon_{eq}} - A_t \exp\left(-B_t\left(\varepsilon_{eq} - \varepsilon_{d0}\right)\right)$$ **(éq 2.1-6)**

$$D_c = 1 - \frac{(1 - A_c)\varepsilon_{d0}}{\varepsilon_{eq}} - A_c \exp\left(-B_c\left(\varepsilon_{eq} - \varepsilon_{d0}\right)\right)$$ **(éq 2.1-7)**

with A_t, A_c, B_t, and B_c, parameters materials to be identified. These parameters make it possible to modulate the shape of the curved post-peak. They are obtained using test and tensile tests of compression.

2.2 Revisited model of Mazars

Although usually employed, the model of Origin of Mazars has gaps in the modeling of the behavior of the concrete during loadings in shearing and bi--compression. A comparison between surfaces of load of the two models is given in Figure 2.2-4.
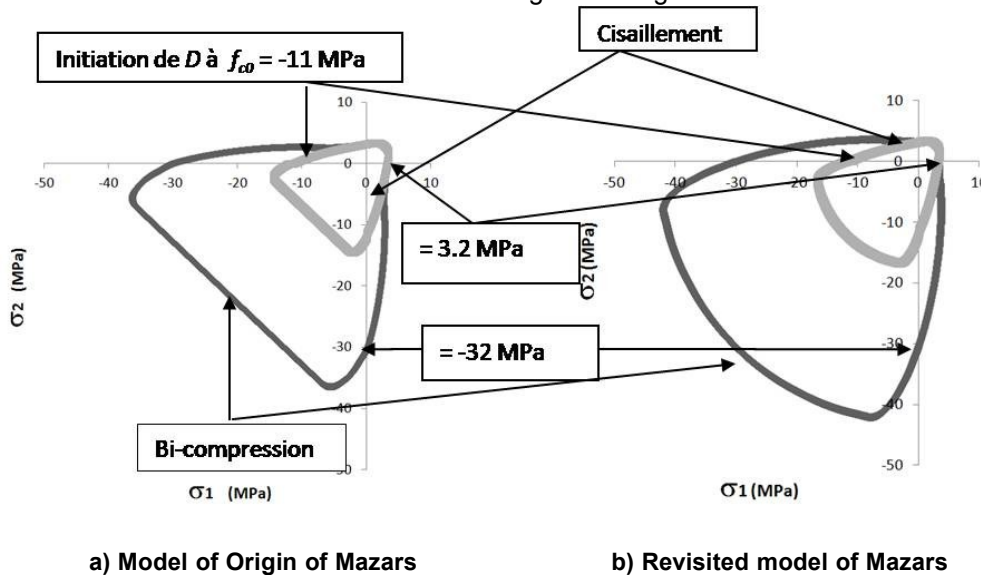

a) **Model of Origin of Mazars**   b) **Revisited model of Mazars**

Figure 2.2-1 : Comparison of surfaces of initiation of damage and rupture of the Mazars models in the plan $\sigma_3 = 0$ and a C30 concrete

Thus, a new formulation is proposed through 2 major modifications:

1. improvement of behaviour in bi--compression,
2. simplification and improvement of behaviour in shearing.

The model of Mazars of origin of the years 1980 1] the resistance of the concrete in bi--compression underestimates largely. The first modification made by the Revisited model thus improves behaviour in bi--compression. This goal is reached by correcting the equivalent deformation when at least a principal constraint is negative, using one variable γ :

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$$\varepsilon_{eq}^{corrigée}=\gamma\,\varepsilon_{eq}=\gamma\,\sqrt{\langle\boldsymbol{\varepsilon}\rangle_+ : \langle\boldsymbol{\varepsilon}\rangle_+}$$ **(éq 2.2-1)**

with:

$$\begin{cases}\gamma=-\dfrac{\sqrt{\sum_i\langle\tilde{\sigma}_i\rangle_-^2}}{\sum_i\langle\tilde{\sigma}_i\rangle_-} & \text{si au moins une contrainte effective est négative}\\[2mm]\gamma=1 & \text{sinon}\end{cases}$$ (limited between 0 and 1) **(éq 2.2-2)**

Lforced effective hasU direction of the mechanics of the damage is defined by:

$$\underline{\tilde{\underline{\sigma}}}=\frac{\underline{\underline{\sigma}}}{1-D}$$ **(éq 2.2-3)**

The definition of < > - is similar to (éq 2.1-3) :

$$\begin{cases}\langle\tilde{\sigma}_i\rangle_-=\tilde{\sigma}_i & \text{si}\quad\tilde{\sigma}_i\leq 0\\\langle\tilde{\sigma}_i\rangle_-=0 & \text{si}\quad\tilde{\sigma}_i>0\end{cases}$$ **(éq 2.2-4)**

where $\tilde{\sigma}_i$ is a principal effective constraint.

The improvement of behaviour in shearing is reached by the introduction of a new internal variable: Y. It corresponds to the maximum reached during the loading of the equivalent deformation. Its initial value Y_0 is ε_{d0}. Y is defined by the following equation:

$$Y=\max\left(\varepsilon_{d0},\max\left(\varepsilon_{eq}^{corrigée}\right)\right)$$ **(éq 2.2-5)**

The function of load is:

$$f=\varepsilon_{eq}^{corrigée}-Y$$ **(éq 2.2-6)**

The evolution of the damage is given by:

$$D=1-\frac{(1-A)Y_0}{Y}-A\exp\left(-B\left(Y-Y_0\right)\right)$$ **(éq 2.2-7)**

In this expression, they are the variables A and B who allow to reproduce the quasi fragile behavior of the concrete in traction and the behavior hammer-hardened in compression. To represent as well as possible the experimental results, the following laws of evolution were selected for A and B :

$$A=A_t\left(2r^2(1-2k)-r(1-4k)\right)+A_c\left(2r^2-3r+1\right)$$ **(éq 2.2-8)**

and

$$B=r^2 B_t+\left(1-r^2\right)B_c$$ **(éq 2.2-9)**

where the expression of r is:

$$r=\frac{\sum_i\langle\tilde{\sigma}_i\rangle_+}{\sum_i|\tilde{\sigma}_i|}$$ **(éq 2.2-10)**

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It appears in these equations a new variable r who informs us about the state of stress. When r is equal to 1 (corresponding to the sector of tractions), the variables A and B are equivalent to the param beings A_t and B_t. Therefore, (éq 2.2-7) is identical to (éq 2.1-6). Conversely, if r is worthless (corresponding to the sector of compressions), then $A = A_c$, $B = B_c$ and (éq 2.2-7) is identical to (éq 2.1-7).

Figure 2.2-2 give in the plan $\sigma_3 = 0$ evolution according to the sign of the principal constraints variables A, B, r and γ.
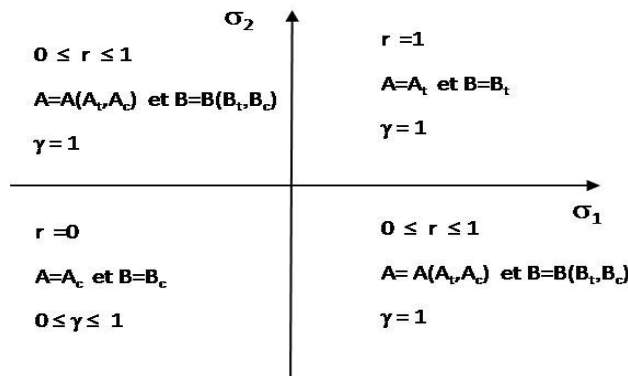

Figure 2.2-2 : Evolution of the variables A, B, r and γ in the plan $\sigma_3 = 0$

In the equation (éq 2.1-6) a new parameter appears: k. It introduces an asymptote with the curve $\sigma - \varepsilon$ in shearing and it is defined by:

$$k = \frac{A_{cisaillement}}{A_t}$$ **(éq 2.2-11)**

where $A_{cisaillement}$ the residual stress in pure shearing defines. It is similar to A_t for this case of loading. The value advised for k is of 0.7. The value of k lower than 1 is very useful the modeling of the effects of friction enters the concrete and the reinforcements in reinforced concrete structures because it induces a residual shear stress. For the value $k = 1$ one finds the behavior of the model of Origin (Figure 2.2-3).
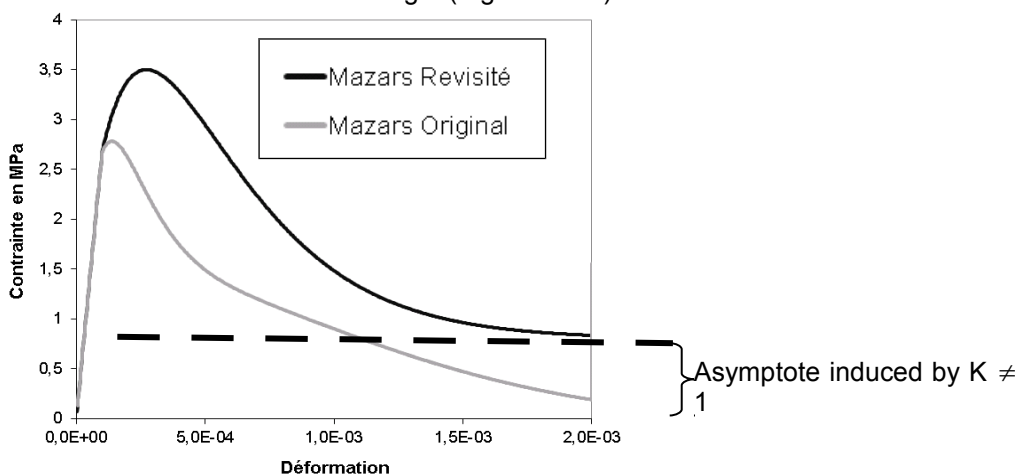

Figure 2.2-3 : Stress-strain curve during a pure shear test on a point of Gauss

The model of Origin under-Estime the resistance of the concrete in pure shearing. This new formulation makes it possible to increase this resistance in pure shearing passing

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from $2.5\,MPa$ with $3.5\,Mpa$ for a C30 concrete. This value depends on those of the parameters materials entered (A_t , A_c , B_t , and B_c).

The local answer of the Revisited model of Mazars under loading successive of traction compression is given by Figure 2.2-4.
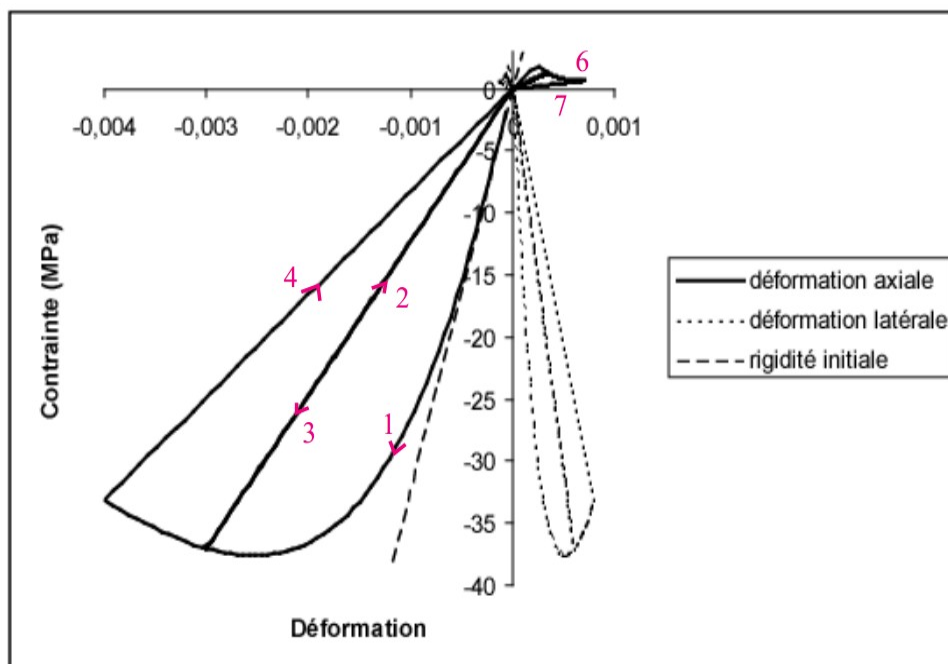

Figure 2.2-4 : Answer stress-strain of the model of Mazars for one request 1D.

Figure 2.2-4 allows to visualize a certain number of characteristics of the model of MAZARS, namely:
•   the damage affects the rigidity of the concrete,
•   there are no unrecoverable deformations,
•   the answers in traction and compression are quite dissymmetrical,

Notice : The models Mazars d' Origine and Revisited do not take into account the unilateral character of the concrete to knowing refermeture of crack at the time of the passage of a state of traction in a state of compression.

3   Identification

In addition to the thermoelastic parameters E, ν, α, the model of Revisited MAZARS fact of intervening 6 parameters material: A_c , B_c , A_t , B_t , ε_{d0}, k .

• ε_{d0} is the threshold of damage. It acts obviously on the constraint with the peak but also on the shape of the curved post-peak. Indeed, the fall of constraint is of as much less brutal than ε_{d0} is small. In general ε_{d0} is understood in 0.5 and $1.5\,10^{-4}$.

Coefficients A and B allow to modulate the shape of the curved post-peak. They are defined by the equations (éq 2.2-8) and (éq 2.2-9) who depend on the parameters of the model of Origin of Mazars (A_t , B_t , A_c and B_c) and of r :

• A introduced a horizontal asymptote which is the axis of ε for $A = 1$ and the horizontal one passing by the peak for $A = 0$ (cf [Figure 3-1]). In the field as of tractions, A is

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equivalent to A_t (and reciprocally in the field of compressions $A = A_c$). In general, A_c lies between 1 and 2. and A_t between 0.7 and 1.

- B according to its value can correspond to a sharp fall of constraint ($B > 10\,000$) or a preliminary phase of increase in constraint followed, after passage by a maximum, of a more or less fast decrease as one can see it on [3-2]. In the field as of tractions, B is equivalent to B_t (and reciprocally in the field of compressions $B = B_c$). In general B_c is understood enters 1000 and 2000 and B_t enter $9\,000$ and $21\,000$.

- k introduced a horizontal asymptote in pure shearing on stress-strain curve if its value is different from 1 for $A_t = 1$, (éq 2.2-11). The advised value is 0.7 .
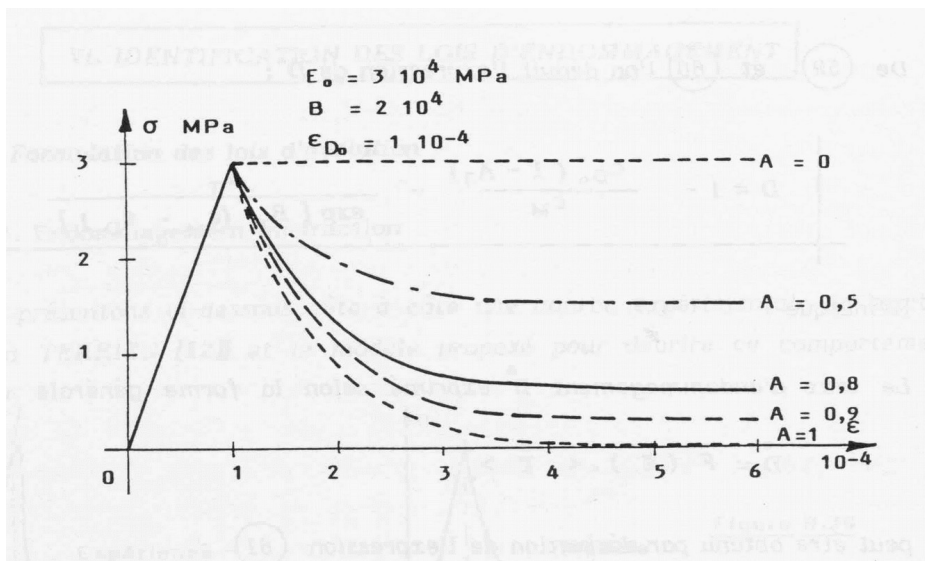

Figure 3-1: Influence of the parameter A_t

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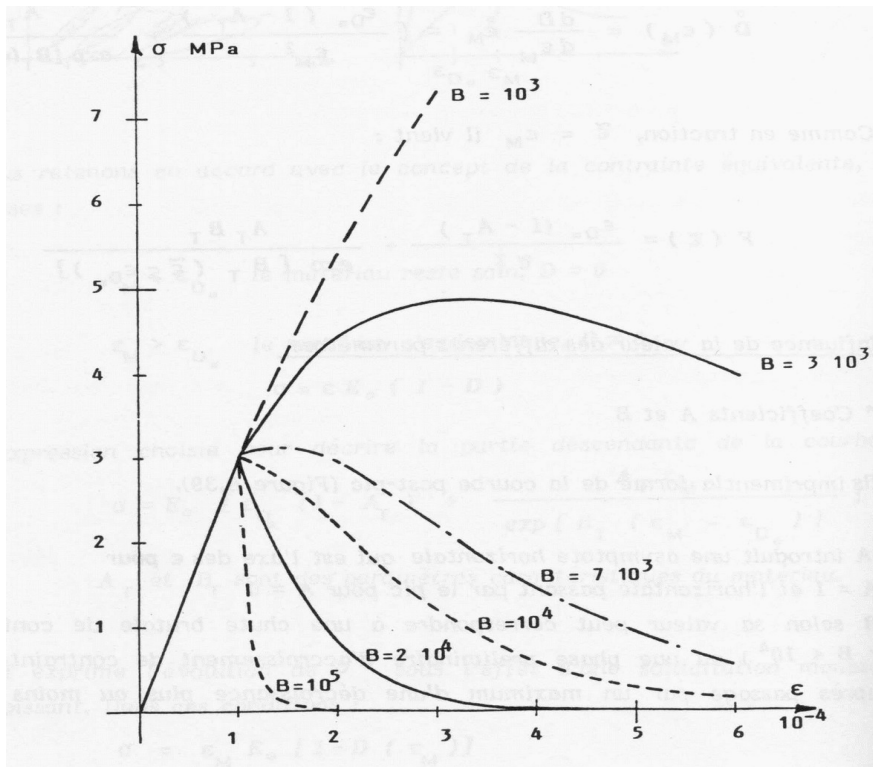
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FigRUE 3-2: Influence of the parameter B_t

A means of obtaining a set of parameters is to have the uniaxial test results in compression and traction (for traction one can use other type of tests, of the "Brazilian" tests of splitting for example).

If one uses the regularization in gradient of deformation (see §1.2), it is recommended to fix the parameters of the law at the same time characteristic length L_c. Some authors (confer [feeding-bottle7]) also suggest gauging L_c in using experimental tests on several sizes of the specimens; indeed, the characteristic length is dependent in keeping with the energy zone of dissipation which could be at the origin of scale effect structural.

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4 Digital resolution

4.1 Evaluation of the internal variable *Y*

The calculation of Y is very simple and follows an explicit diagram. The stages are the following ones:

- Calculation of the elastic strain and thermal
- Calculation of the principal elastic constraints and evaluation of γ (éq 2.2-2).
- Calculation of the equivalent deformation ((éq 2.1-2) and (éq 2.2-1)).
- Calculation of the variables r, A and B
- Calculation of the internal variable Y *(éq 2.2-5)*.

If $Y \leq Y^-$ then $Y^+ = Y^-$.

If $Y > Y^-$ *then* $Y^+ = Y$.

Note: Currently the variable stored during calculations is ε_{eq} in order not to modify them existing couplings with UMLV. A condition on the strictly increasing evolution of the damage allows this simplification if γ vary.

4.2 Evaluation of the damage

The damage is calculated in all the cases with the equation (éq 2.2-7).

$$D = 1 - \frac{(1-A)Y_0}{Y} - A \exp\left(-B(Y - Y_0)\right)$$ (éq 2.2-7)

It is important to specify that we impose on D to be ranging between 0 and 1 because it is possible to have values outwards this framing following the choice of the parameters materials as for the model of origin.

4.3 Calculation of the constraint

After evaluation of D, we calculate simply:

$$\sigma = (1-D)\mathbf{A}\,\varepsilon^e$$ **(éq 4.3-1)**

4.4 Calculation of the tangent matrix

One of the disadvantage of the model of Mazars is the absence of tangent matrix. It is not possible to calculate this matrix because of the use of the MacCauley operator in the calculation of the equivalent deformation (éq 2.1-2), of γ (éq 2.2-2) and r. However it is possible to use a radial approximation during loading.

Us let us seek the tensor $\mathbf{M}$ such as $\dot{\sigma} = \mathbf{M}\dot{\varepsilon}$ knowing that $\sigma = (1-D)\mathbf{A}\varepsilon$. The matrix is thus the sum of two terms, one with constant damage, the other due to the evolution of the damage:

$$\dot{\sigma} = (1-D)\mathbf{A}\dot{\varepsilon} - \mathbf{A}\varepsilon\dot{D}$$ **(éq 4.4-1)**

The first term is easy, it acts of the operator of Hooke, multiplied by the factor $1-D$.

The second requires the evaluation of the increment of damage $\dot{D}$.

If a radial loading is imposed, variables γ, r, A and B are constant. While posing:

$$\dot{D} = \frac{\partial D}{\partial Y} \frac{\partial Y}{\partial(\gamma\,\varepsilon_{eq})} \frac{\partial(\gamma\,\varepsilon_{eq})}{\partial\varepsilon}\dot{\varepsilon}$$ **(éq 4.4-2)**

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$$\frac{\partial Y}{\partial (\gamma \varepsilon_{eq})} \frac{\partial (\gamma \varepsilon_{eq})}{\partial \varepsilon} = \frac{\gamma \langle \varepsilon \rangle_+}{\varepsilon_{eq}}$$ **(éq 4.4-3)**

Under this condition of radial loading, the increment of deformation is written:

$$\dot{D} = \left[\frac{(1-A)Y_0}{Y^2} + AB \exp(B(Y-Y_0)) \right] \frac{\gamma \langle \varepsilon \rangle_+}{\varepsilon_{eq}} \dot{\varepsilon}$$ **(éq 4.4-4)**

Note:

1. *Being given made simplifications, in the case general the tangent matrix is not consistent. Also, it can happen that the reactualization of the tangent matrix during iterations of Newton does not help with convergence. In this case, it is enough to use only the secant matrix while imposing* STAT_NON_LINE (NEWTON = _F (REAC_ITER =0)).

2. *In the case general, the tangent matrix is not-symmetrical. It is possible to do it thanks to the keyword* SOLVEUR=_F (SYME = 'YES') *of* STAT_NON_LINE.

3. *Concerning the nonlocal approach, the treatment of the boundary conditions is such as one could be brought, in the case of symmetrical structures, to treat the calculation of the whole of the structure and not of the "representative" part (cf [R5.04.02]).*

4. *The analytical expression of the tangent matrix is valid only for radial loadings (* $dr = d\gamma = 0$ *). In the other cases, the quadratic convergence of the method is not guaranteed any more.*

4.5 Stored internal variables

We indicate in the table according to the internal variables stored in each point of Gauss for the model of MAZARS :

Internal variable	Physical direction
$V1$	D : variable of damage
$V2$	indicator of damage (0 so elastic, 1 if damaged i.e. as soon as D is not null any more)
$V3$	$Tmax$: temperature θ maximum reached at the point of gauss
$V4$	$\varepsilon_{eq} = \sqrt{\langle \varepsilon \rangle_+ : \langle \varepsilon \rangle_+}$ equivalent deformation

Table 4.5-1: Stored internal variables.

5 Features and checking

The law of behavior MAZARS, keyword BEHAVIOR of STAT_NON_LINE, associated material MAZARS is usable in Code_Aster with various modelings:

- classical version: 3D, D_PLAN, AXIS, C_PLAN (established analytical formulation, not to use the method DEBORST)
- not-local version: 3D_GRAD_EPSI, D_PLAN_GRAD_EPSI, C_PLAN_GRAD_EPSI,
- coupled with the models of THHM (confer [R7.01.11]).

The law of MAZARS can be coupled with the model of creep BETON_UMLV_FP (confer [R7.01.06]) via the keyword KIT_DDI. This is true as well for the local version as thelocal one.

The law of behavior is checked by the following tests:

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COMP007b	[V6.07.107]	Test of compression-dilation for study of the coupling thermics-cracking
HSNV129	[V7.22.129]	Test of compression-dilation for study of the coupling thermics-cracking
SSLA103	[V3.06.103]	Calculation of the withdrawal of desiccation and the endogenous withdrawal on a cylinder
SSNP113	[V6.03.113]	Rotation of the principal constraints (law of MAZARS)
SSNP161	[V6.03.161]	Biaxial tests of Kupfer
SSNV157	[V6.04.157]	Test of the method of delocalization per regularization of the deformation on a variable bar of section in traction
SSNV169	[V6.04.169]	Coupling creep-damage
WTNV121	[V7.31.121]	Damping of the concrete with a law of damage

Table 5-1 : existing CAS-tests

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7 History of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
6.4	S.MICHEL-PONNELLE	Initial text
7.4	S.MICHEL-PONNELLE	
9.4	S.MICHEL-PONNELLE, Marina BOTTONI	Addition of the coupling UMLV-MAZARS + internal card 11150 + 4th variable
11.0	Marina BOTTONI	
11.2	François HAMON	Reformulation of the Mazars model