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1. Abstract

The aim of this job is to compare FEA results with theory results of simple problems in structural mechanic.

2. Thick cylinder under internal pressure

(File: TC1_calculix.inp; TC1Co_calculix.inp, TC1Asym_calculix.inp)

A thick cylinder of 200 mm inner diameter and 50 mm thick, is subjected to an internal pressure of 8 MPa.

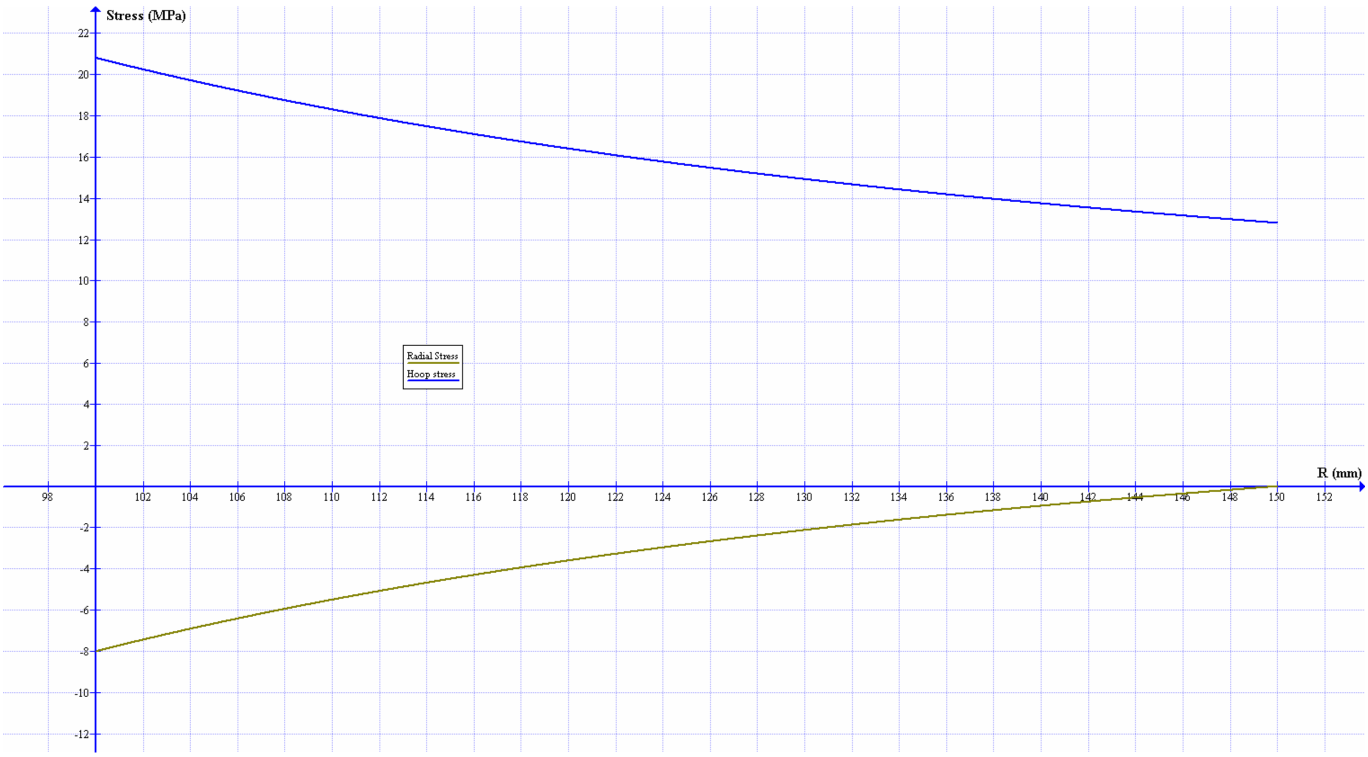


fig. 1: Radial and hoop stress from theory

Model description summary	
Material	Elastic with elastic modulus of 2e5 MPa and Poisson's ratio of 0,3
Type of analysis	Plane stress formulation
Type of elements (Calculix elements library)	CPS8

From the solver the radial and the hoop stresses are depicted on next figures.

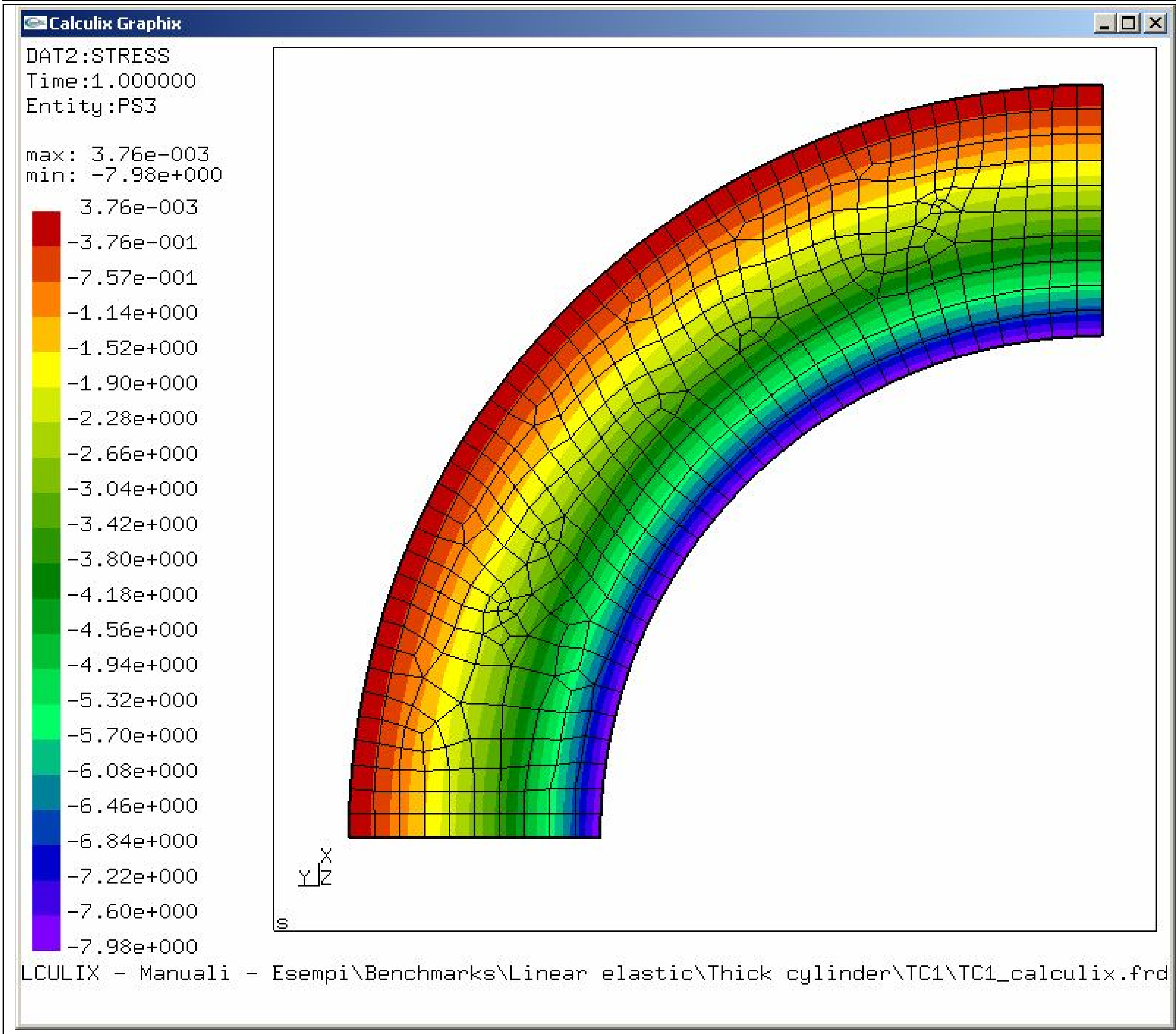


fig. 2: Radial stress

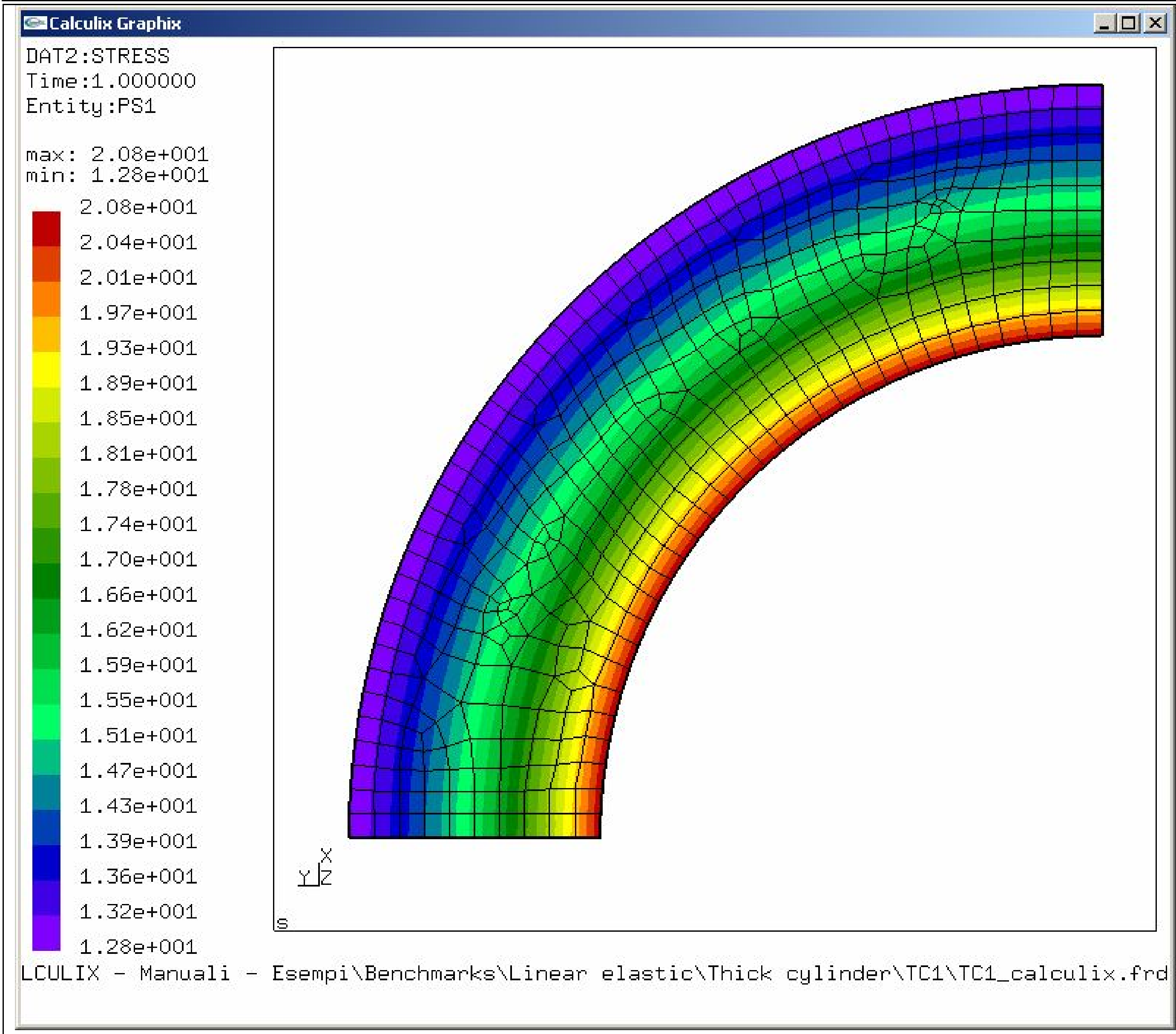


fig. 3: Hoop stress

On next table the results form theory and from the solver.

R (mm)	Radial stress		Hoop stress	
	Theory	CCX	Theory	CCX
100	-8,00	-7,96	20,80	20,82
110	-5,50	-5,47	18,30	18,33
120	-3,60	-3,58	16,40	16,42
130	-2,12	-2,11	14,92	14,94
140	-0,95	-0,94	13,75	13,76
150	0	0,0033	12,80	12,80

With coarse mesh the results are on next table.

R (mm)	Radial stress		Hoop stress	
	Theory	CCX	Theory	CCX
100	-8,00	-7,74	20,80	20,92
112,5	-4,98	-4,91	17,78	18,60
125	-2,82	-2,64	15,62	15,77
137,5	-1,22	-1,14	14,02	14,13
150	0	0,012	12,80	12,82

For the same scope is possible utilize CAX8 elements for axis symmetric study. Next pictures describe the radial (SXX) and hoop stress (SZZ).

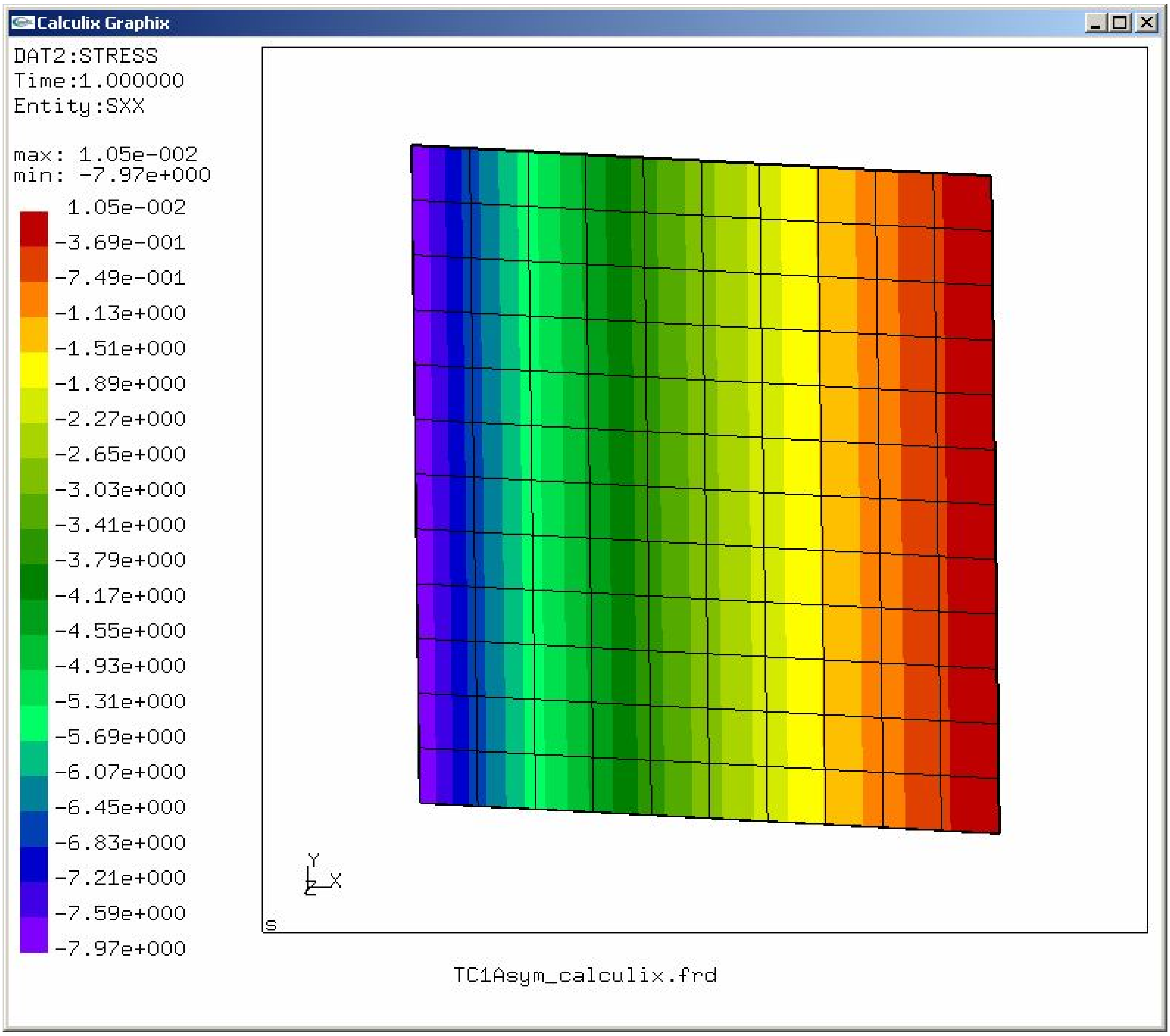


fig. 4

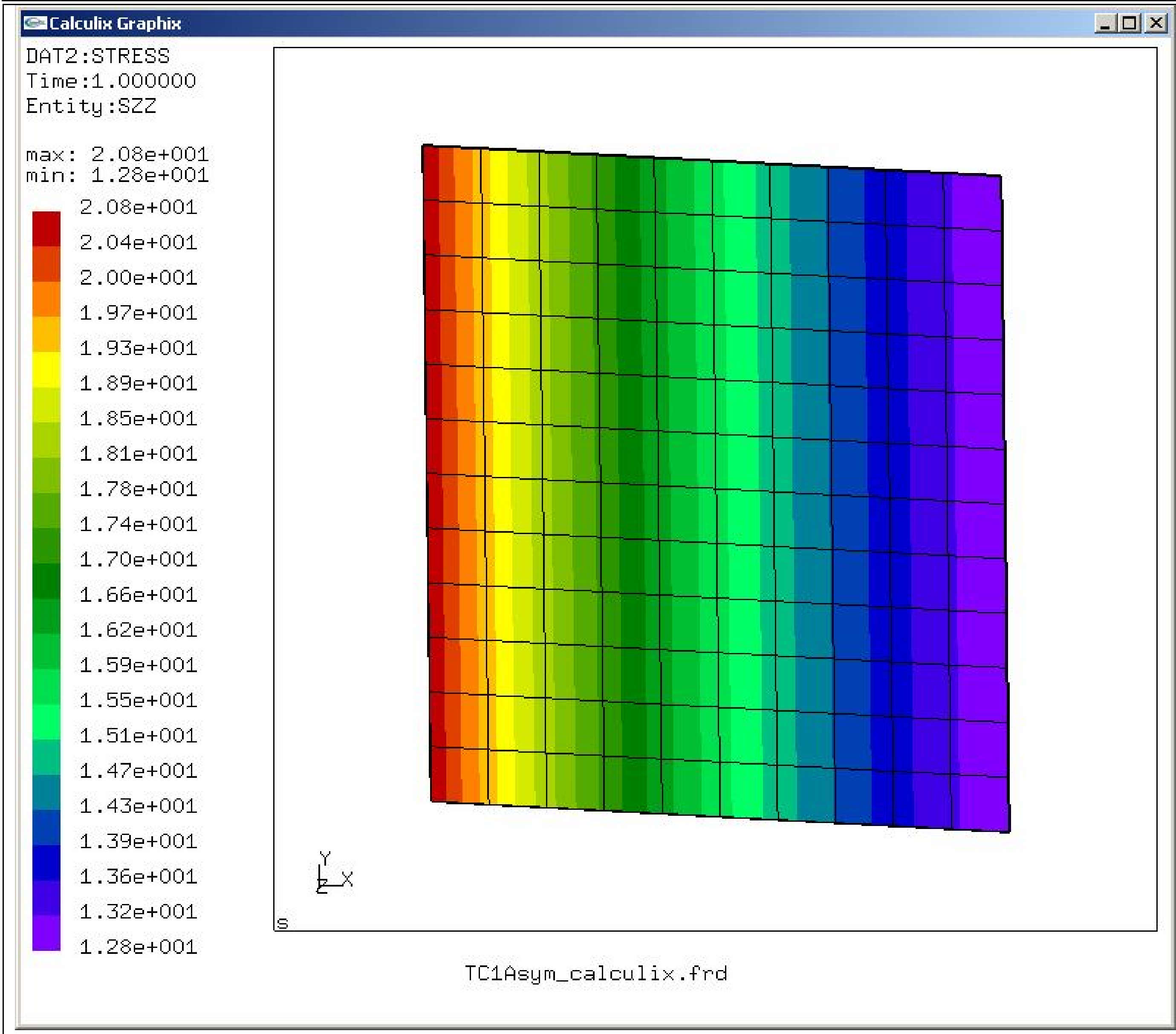


fig. 5

The results are the same.

3. Rotating thick cylinder

(File: TC2.inp)

A cylinder, 200 mm outer diameter and 50 mm thick, rotates at 2400 r.p.m. The density equals 7600 kg/m^3 and the Poisson's ratio equals 0,3.

Theoretical and CCX values of maximum hoop stress and radial stress are respectively:

Theory	CCX
$\sigma_h = 4.286\ MPa$	$\sigma_h = 4.21\ MPa$
$\sigma_r = 0.514\ MPa$	$\sigma_r = 0.531\ MPa$

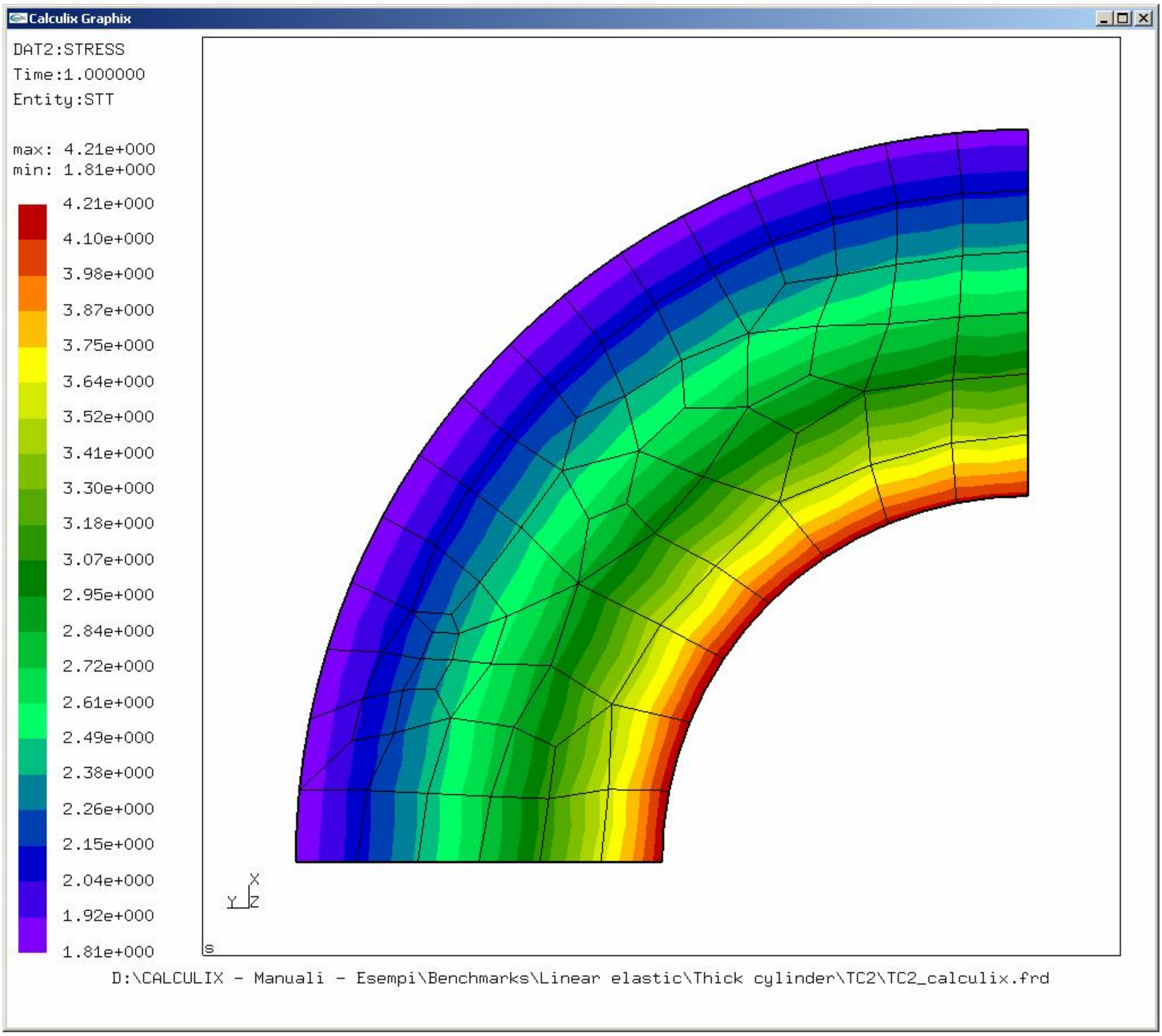


fig. 6: Hoop stress

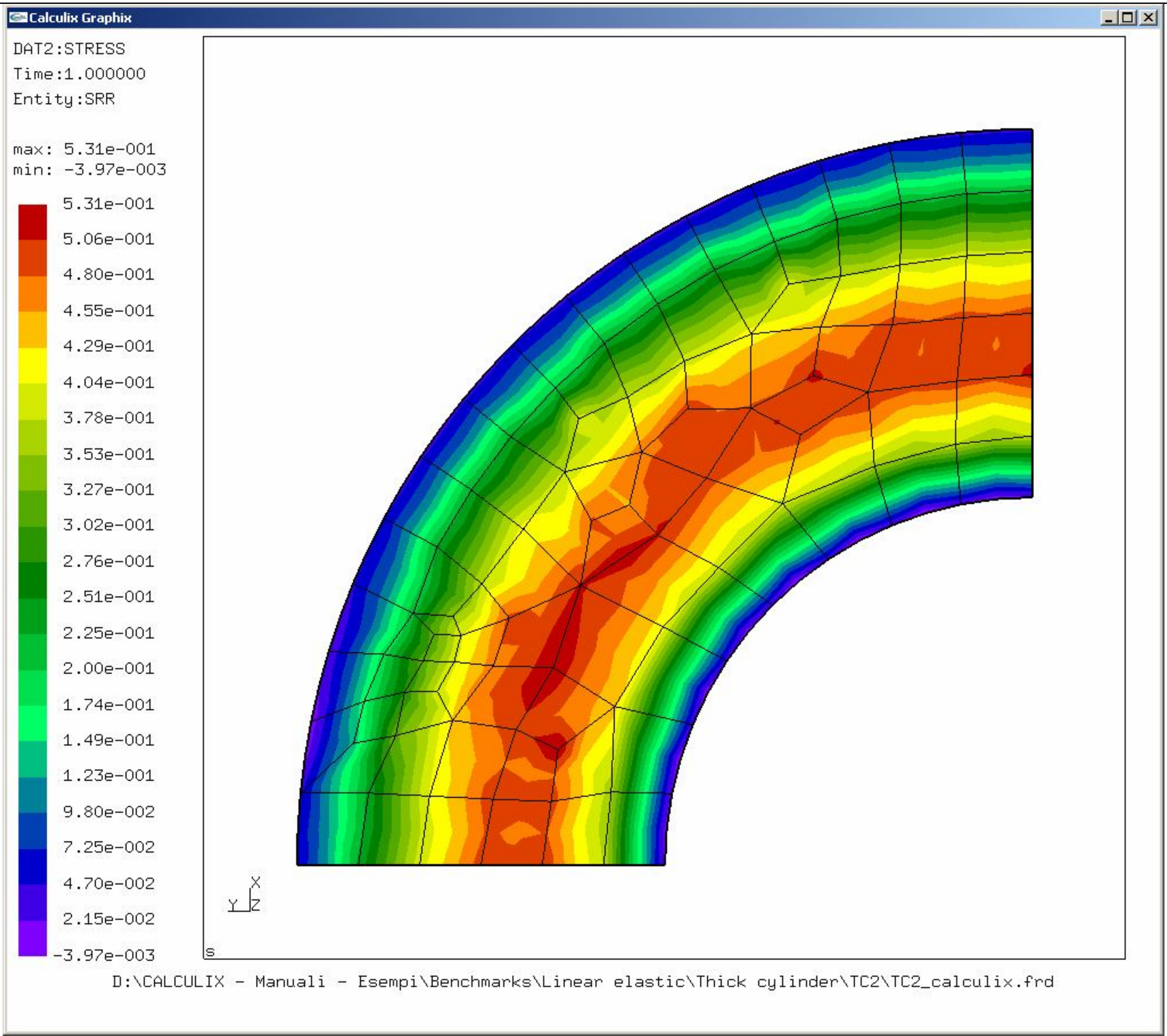


fig. 7: Radial stress

Note: Version 2.3 of CCX for Windows sometimes gives problems with the card *TRANSFORM and so I had considered a quarter of model instead an half or 1/6

NOTE2: Figures 4 and 5 show the hoop stress and the radial stress in cylindrical coordinates. It’s possible in CGX typing “trfm cyl z”

4. Flat circular plate under pressure: clamped edge

(File: PL1asym.inp; PL1tetra.inp; PL1hexa.inp)

A flat circular plate of 120 mm diameter and 6,35 mm thick, is subjected to an uniform pressure of 0,345 MPa. Elastic modulus of 2e5 MPa and Poisson’s ratio of 0,3

The theoretical value of the maximum deflection from theory equals 0,0149 mm if the edge is clamped.

Theory	CCX (CAX8)	CCX (C3D10)	CCX (C3D20)
0,0149 mm	0,0155 mm	0,0153 mm	0,0155 mm

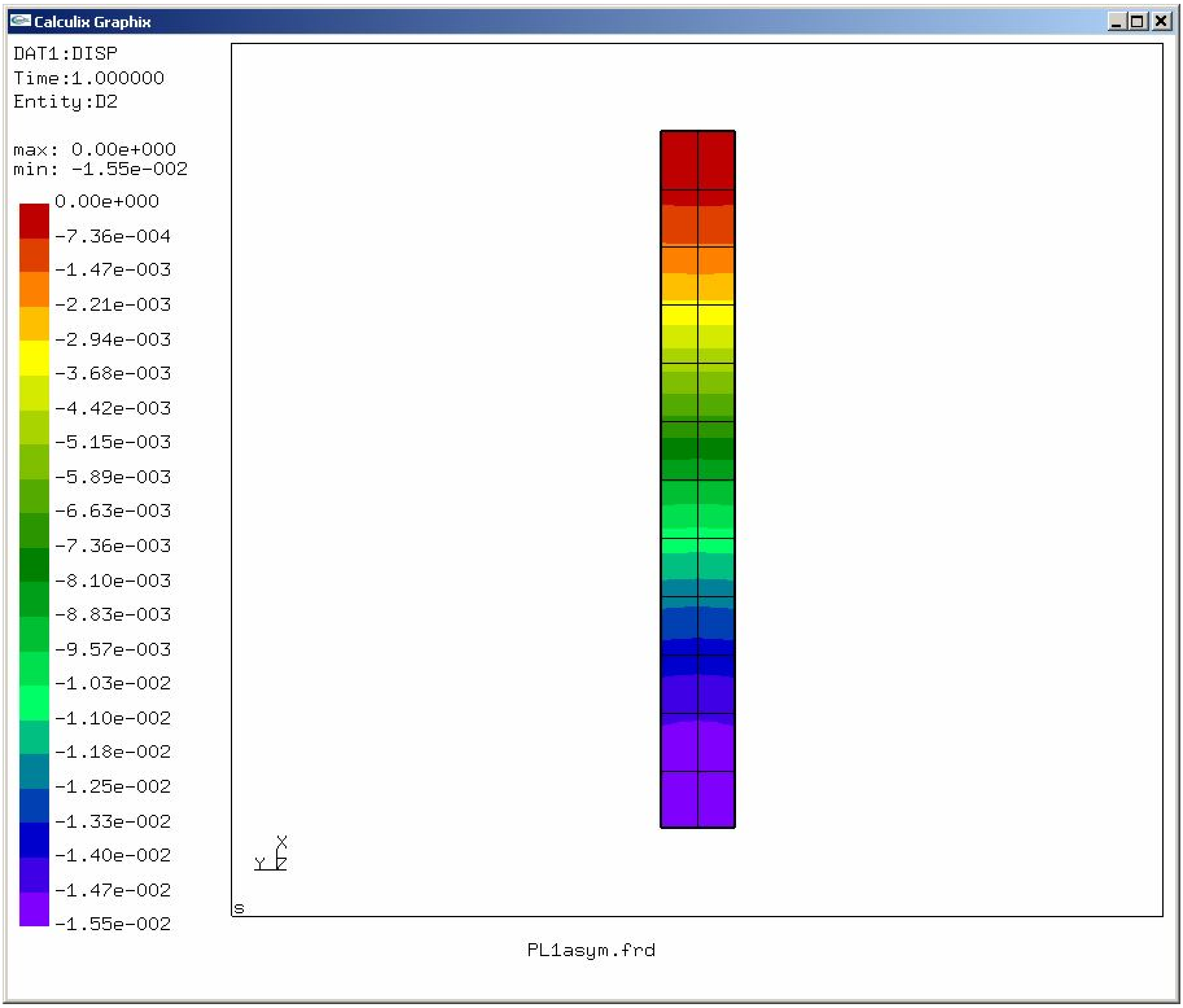


fig. 8: Deflection with CAX8 elements

With 3D solid elements (C3D8) the result is shown on next figure.

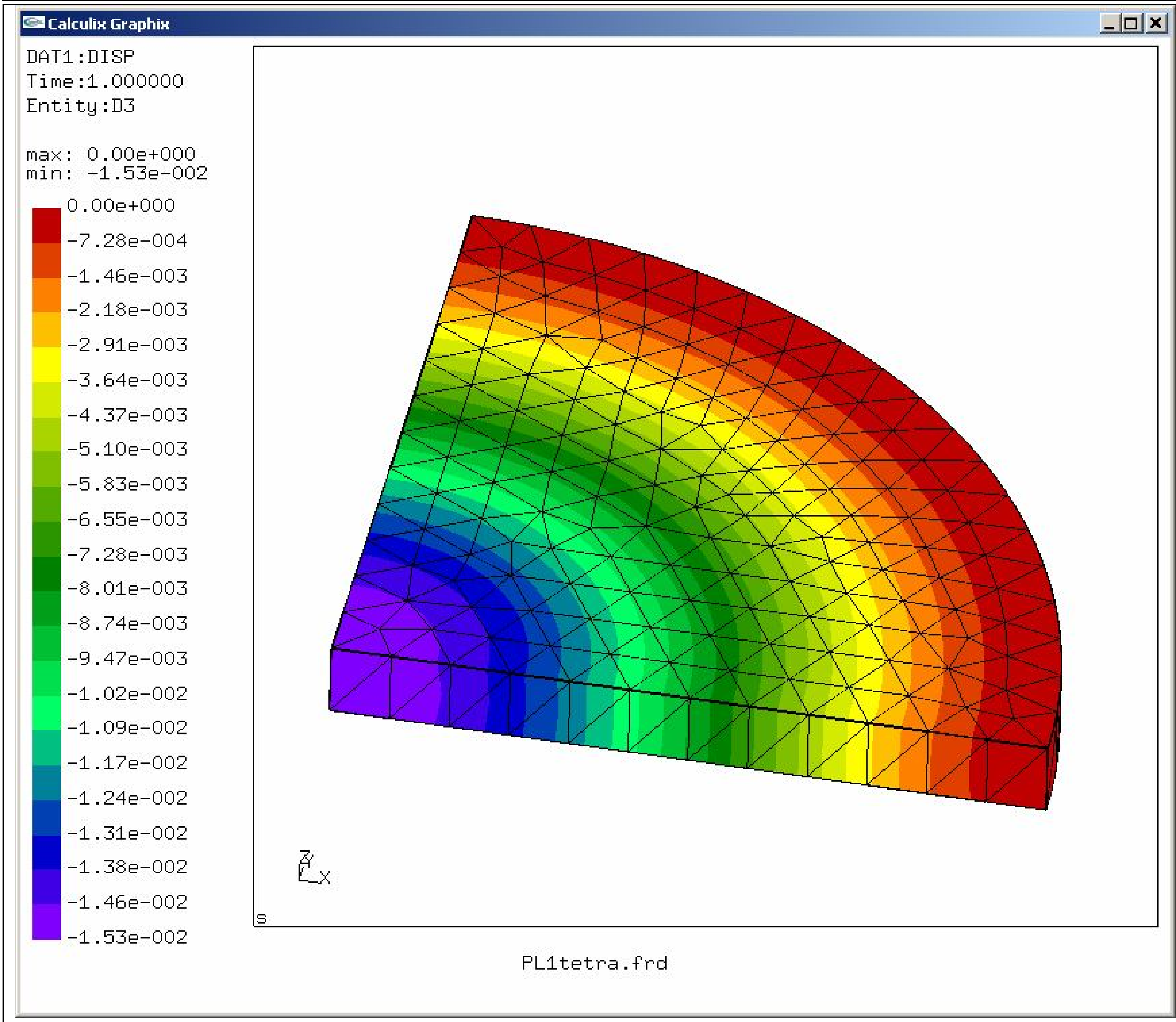


fig. 9: Deflection with C3D10 elements

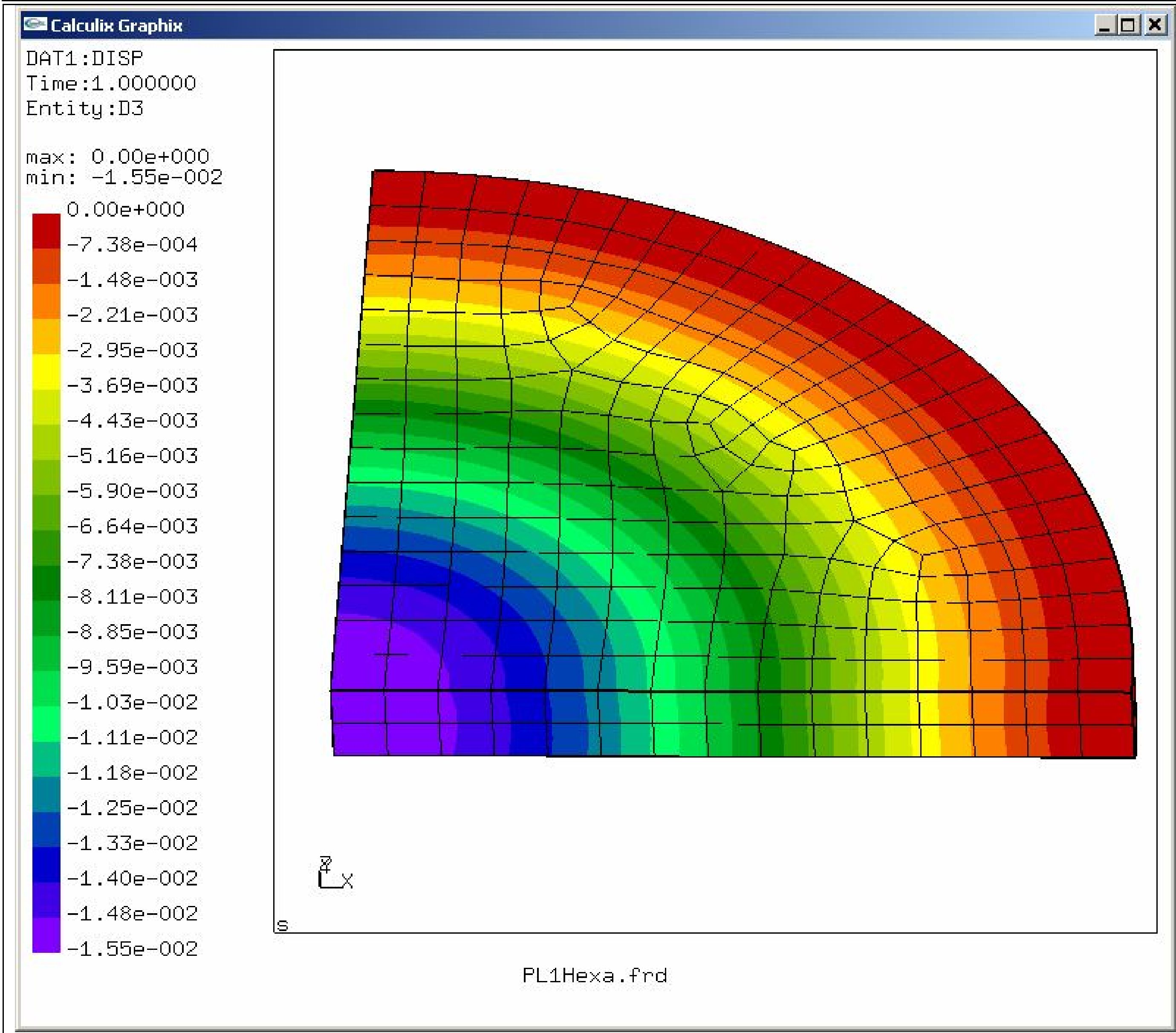


fig. 10: Deflection with C3D20 elements

5. Flat circular plate under pressure: simply supported edge

(File: PL2asym.inp; PL2tetra.inp; PL2hexa.inp)

A flat circular plate of 100 mm diameter and 5 mm thick, is subjected to an uniform pressure of 0,35 MPa. The edge is simply supported. Elastic modulus of 2e5 MPa and Poisson’s ratio of 0,3.

The theoretical value of the maximum deflection from theory equals 0,061 mm if the edge is simply supported.

Theory	CCX (CAX8)	CCX (C3D10)	CCX (C3D20)
Deflection at the center			
0,061 mm	0,0616 mm	0,0615 mm	0,0616 mm
Maximum radial stress			
43,3125 MPa	43,8 MPa	43,3 MPa	43,4 MPa

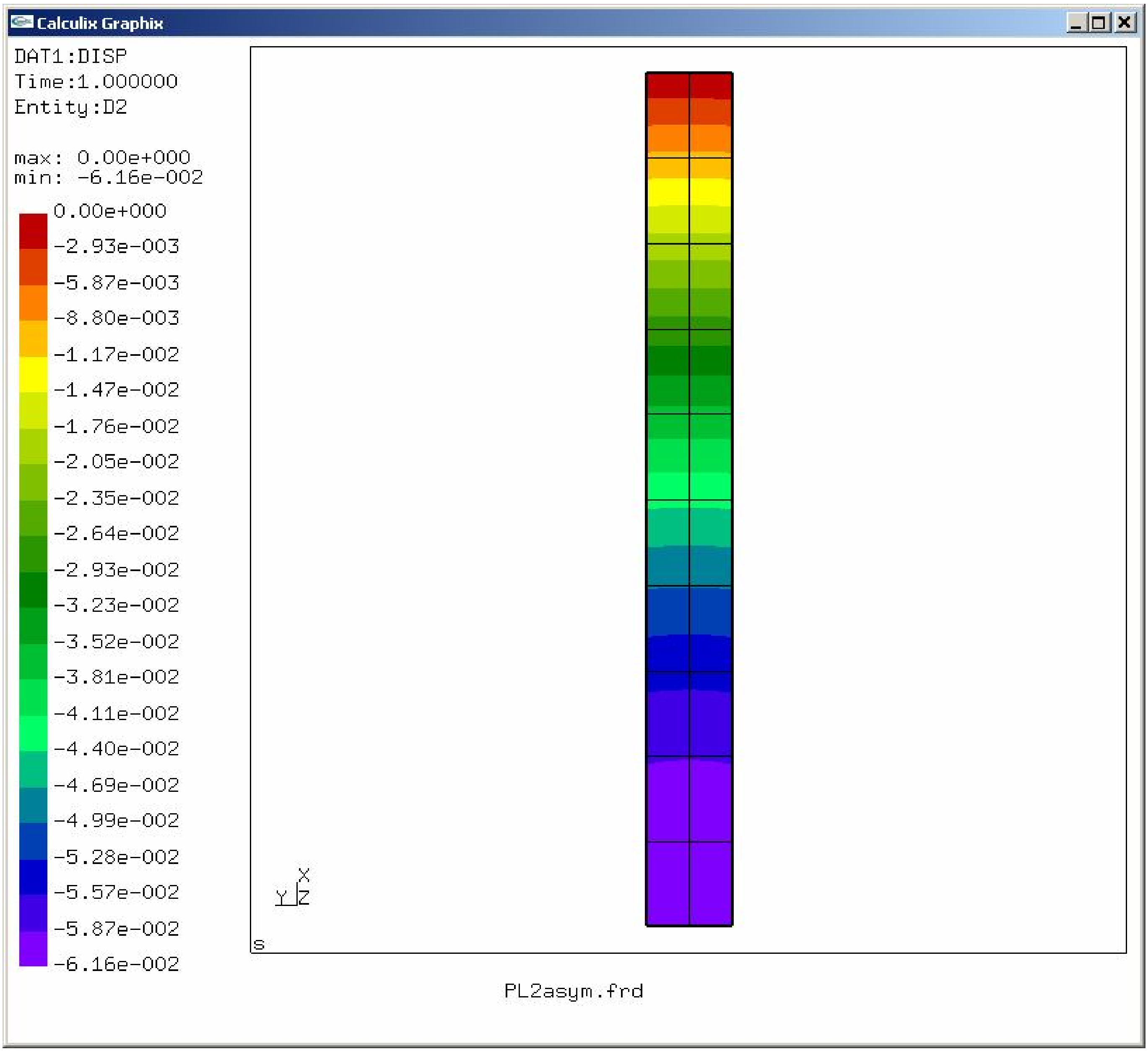


fig. 11: Deflection with CAX8 elements

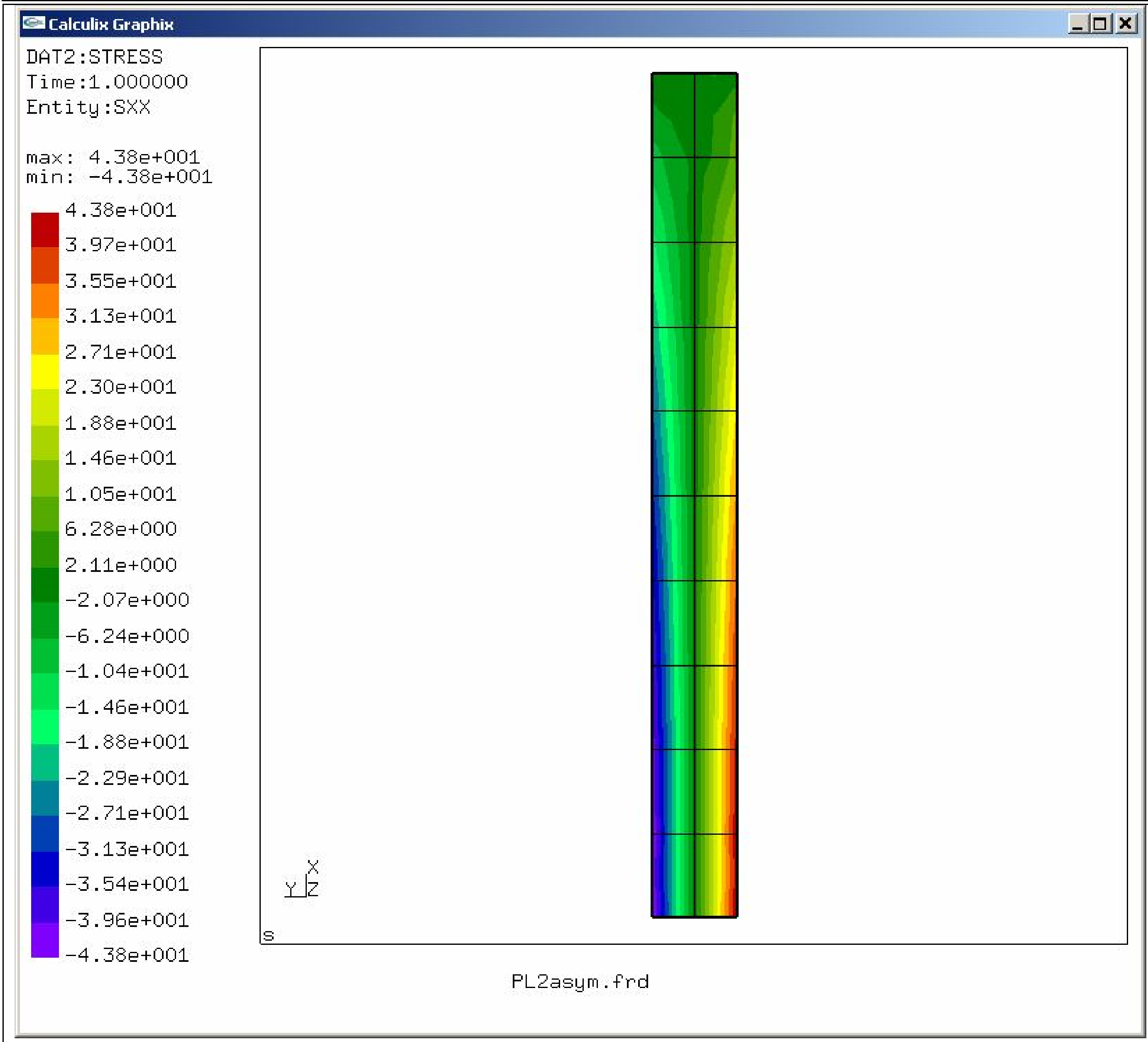


fig. 12: Radial stress with CAX8 elements

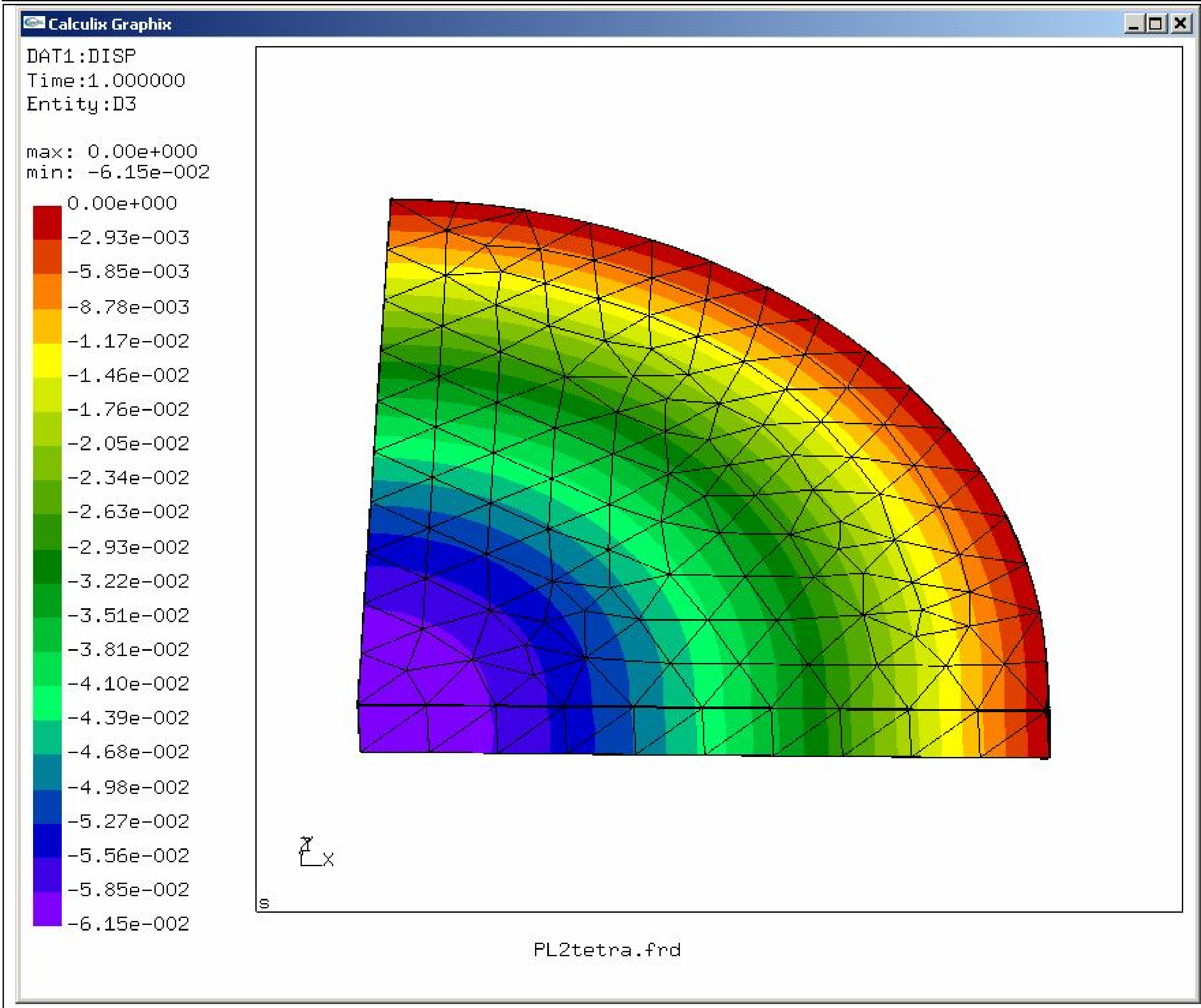


fig. 13: Deflection with C3D10 elements

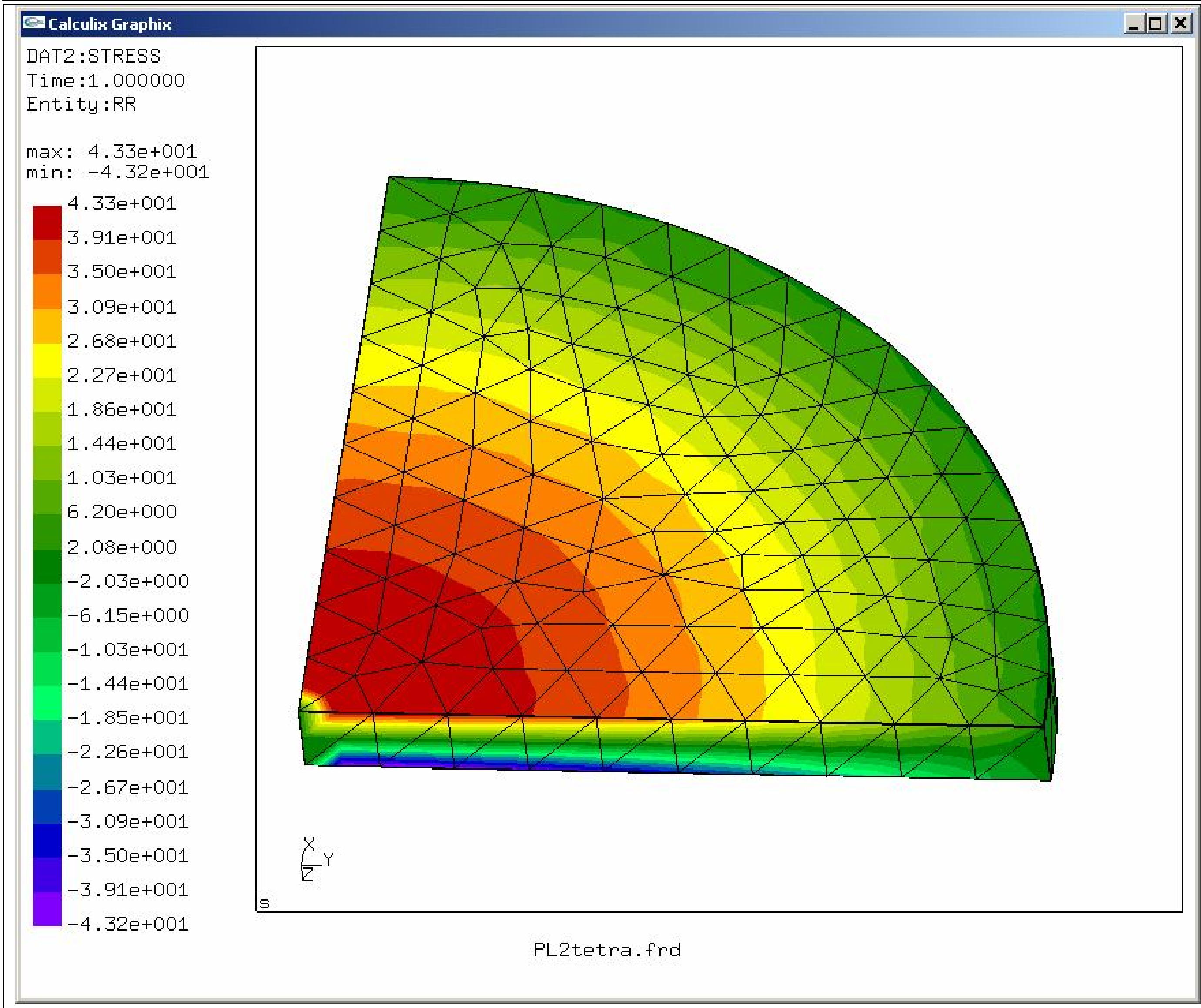


fig. 14: Radial stress with C3D10 elements

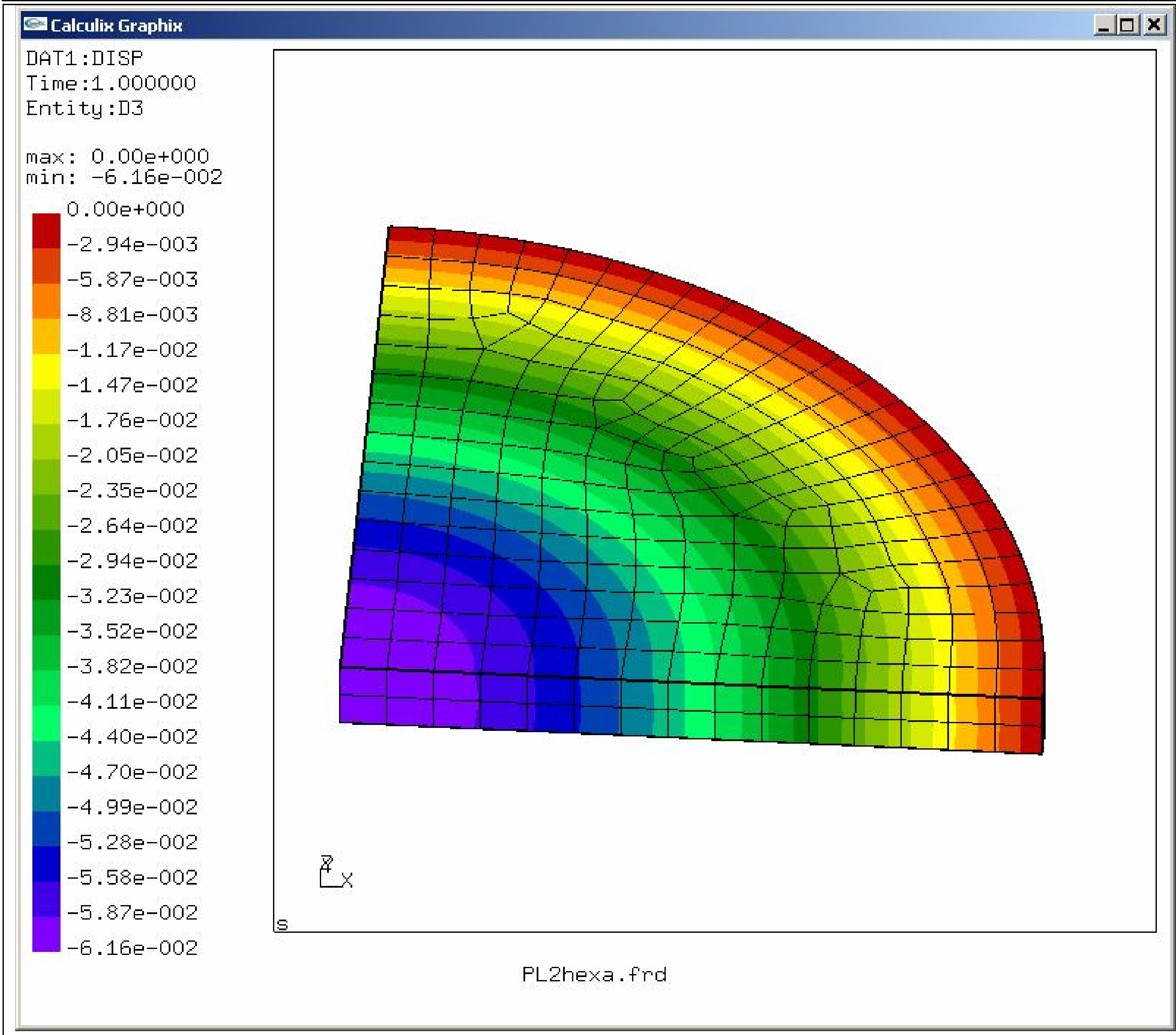


fig. 15: Deflection with C3D20 elements

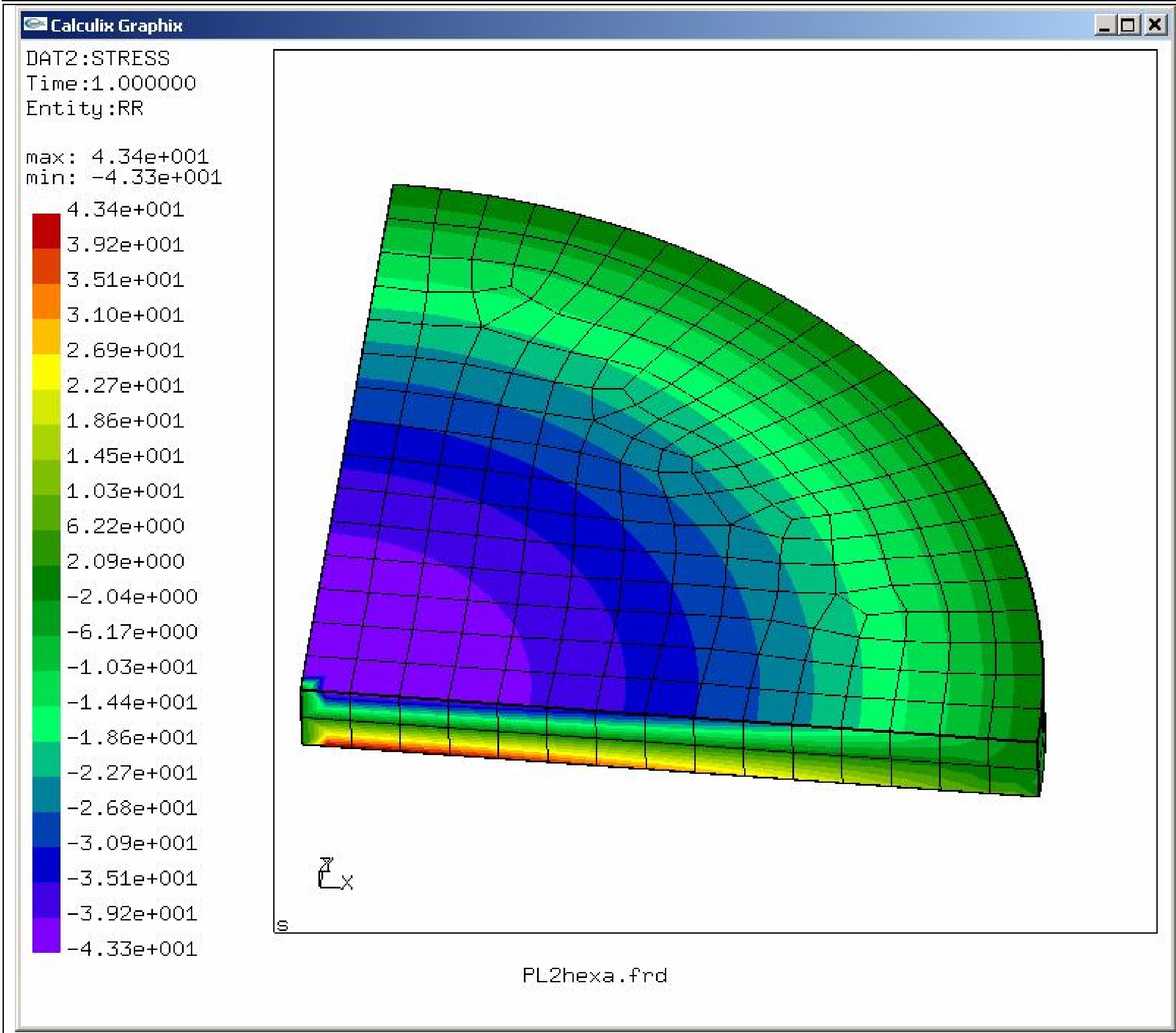


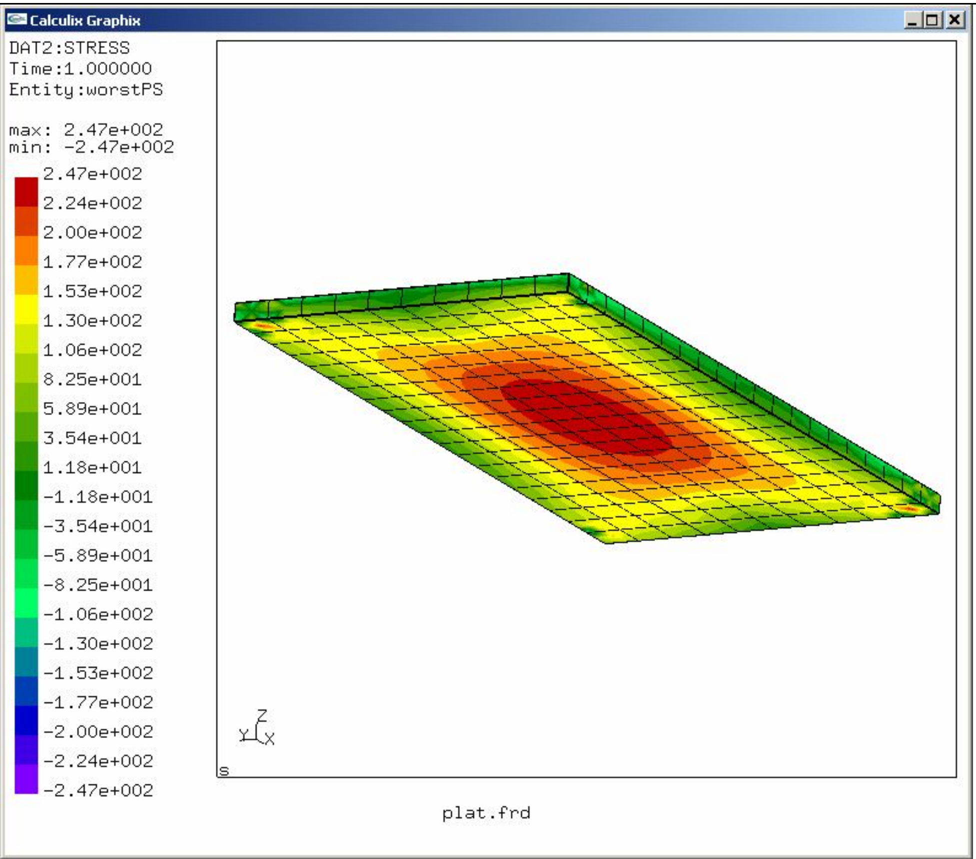
fig. 16: Radial stress with C3D20 elements

6. Rectangular plate simply supported under uniform pressure

(File: PL3.inp)

A rectangular plate (100x50x2,5 mm) simply supported at the four edges is subjected to a constant pressure of 1MPa.
E =200000 MPa; $\nu = 0{,}3$

Theory	CCX (S8)
Max σ_1 (at the center)	
244,08 MPa	±247,9 MPa
Maximum displacement (at the center)	
0,222 mm	0,225 mm



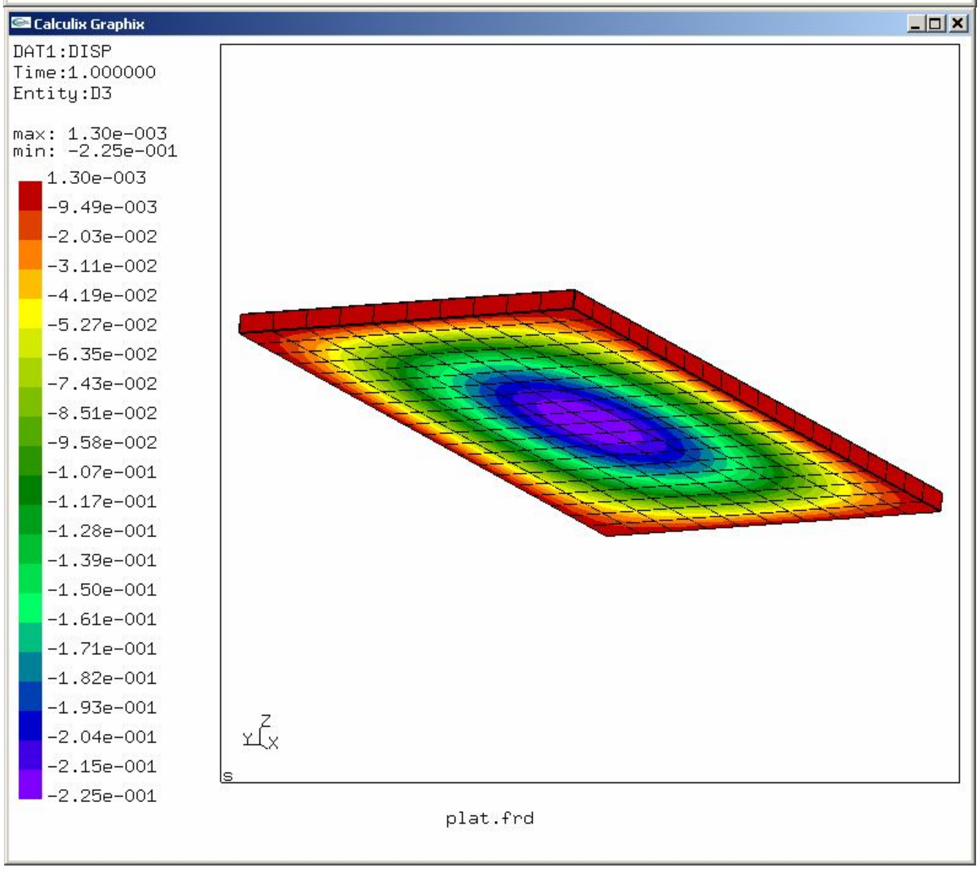


fig. 17: Maximum and minimum principal stresses and z displacement

7. Toroidal shell under internal pressure

A toroidal pressure vessel made of steel, is subjected to an internal pressure of 10 MPa and it has the following dimension referred to next picture:

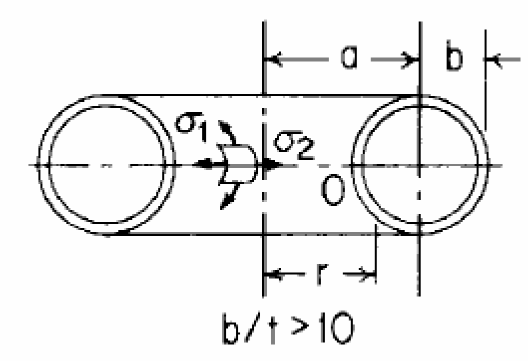
$a = 500$ mm

$b = 100$ mm

$t = 5$ mm

$E = 200000$ MPa

$\nu = 0{,}3$



Theory	CCX (CAX8)
Tangential stress: Max σ_1 (b = 100 mm)	
225 MPa	225,9 MPa
Axial stress (b = 100 mm)	
100 MPa	85,3 ÷ 101,1 MPa

Theory	CCX (CAX8)
Radius variation Point A (r = 400 mm)	
+0,065	+0,062 mm
Radius variation Point B (r = 600 mm)	
+0,135	+0,121 mm

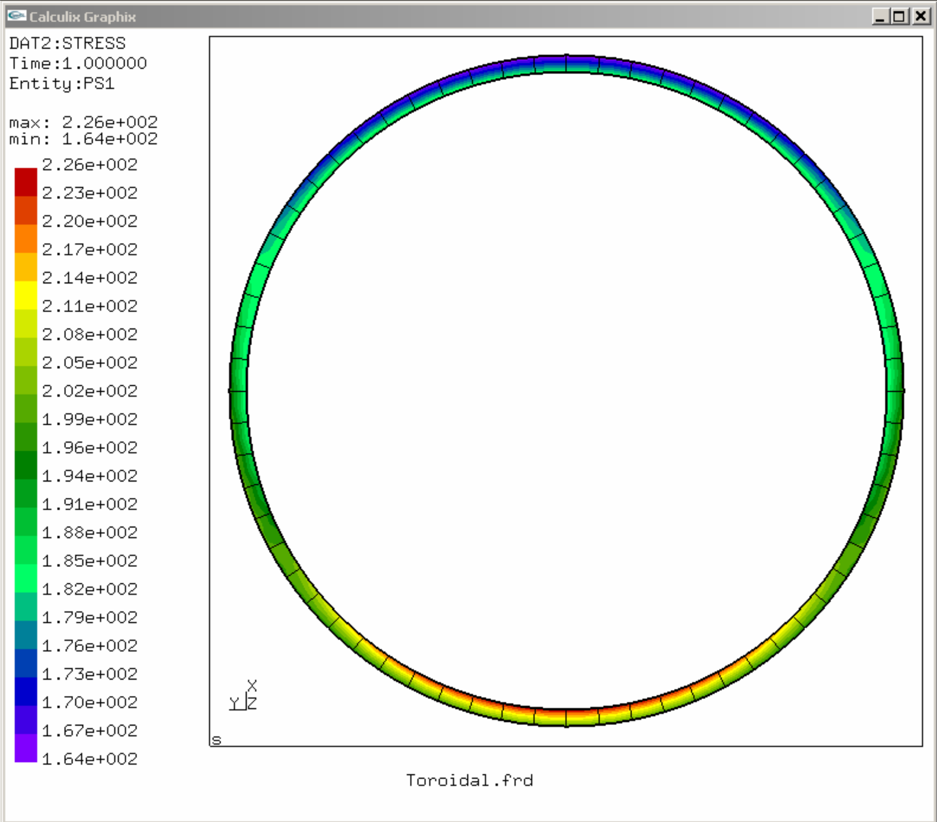


fig. 18: Tangential stress

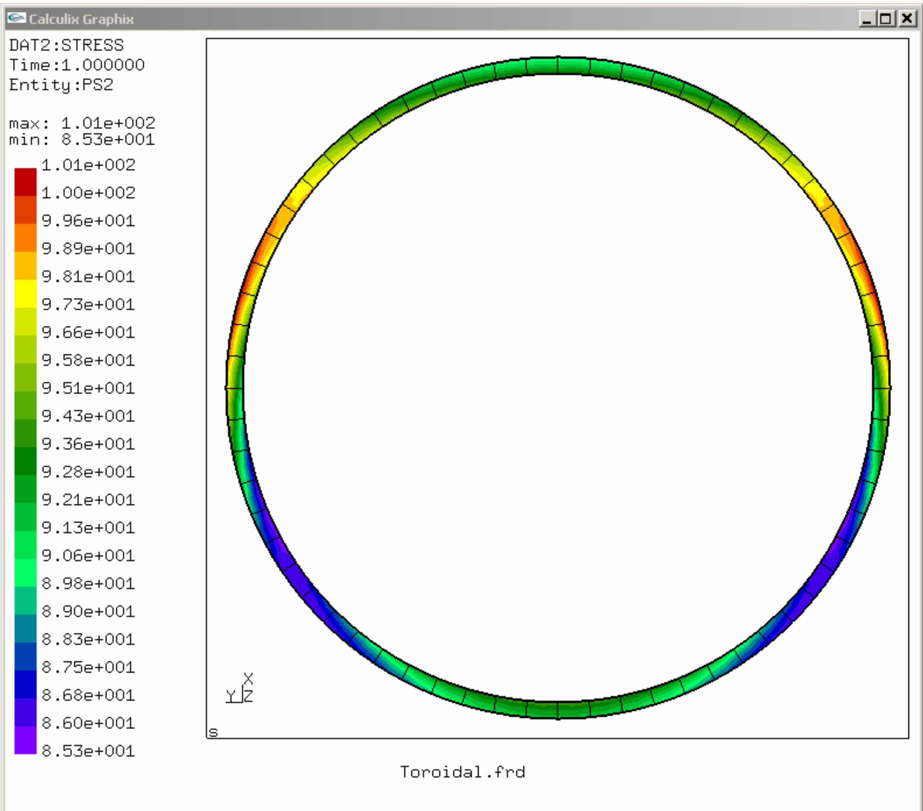


fig. 19: Radial stress

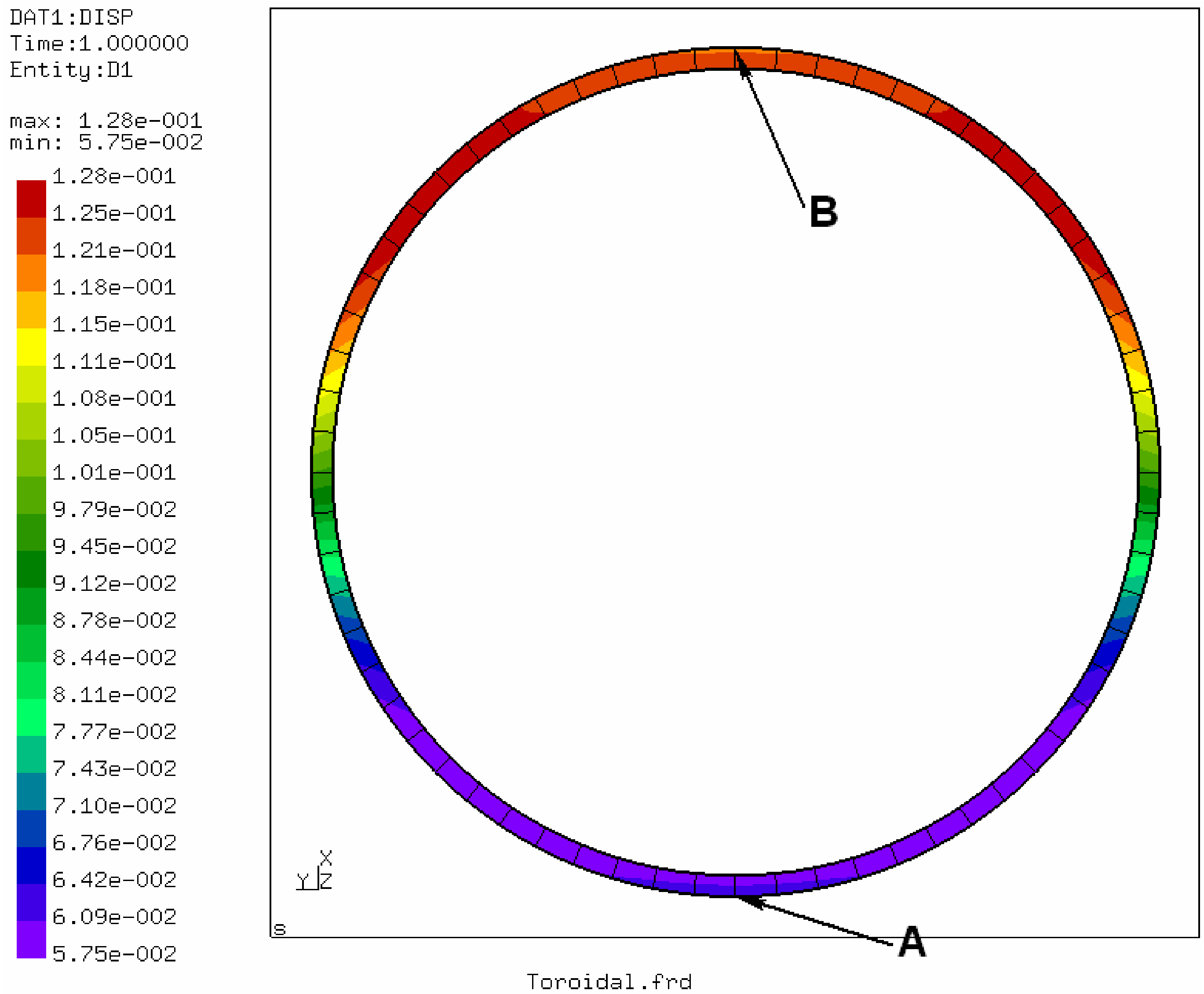


fig. 20: Displacements on x direction

8. Curved beam

A curved beam (section 200 th 90) of inner radius of 300 mm and outer radius of 500 mm is subjected to a traction force of 300 kN. Elastic modulus of 2e5 MPa and Poisson's ratio of 0,3.

Theory	CCX (CPE6)	CCX (CPS6)	CCX (CPE8)	CCX (CPS8)	CCX (C3D10)
Maximum principal stress MPa					
257	260,4	262,5	262,2	265,2	261,7
Minimum principal stress MPa					
-154	-156,5	-157,5	-158,1	-160,1	-157,1

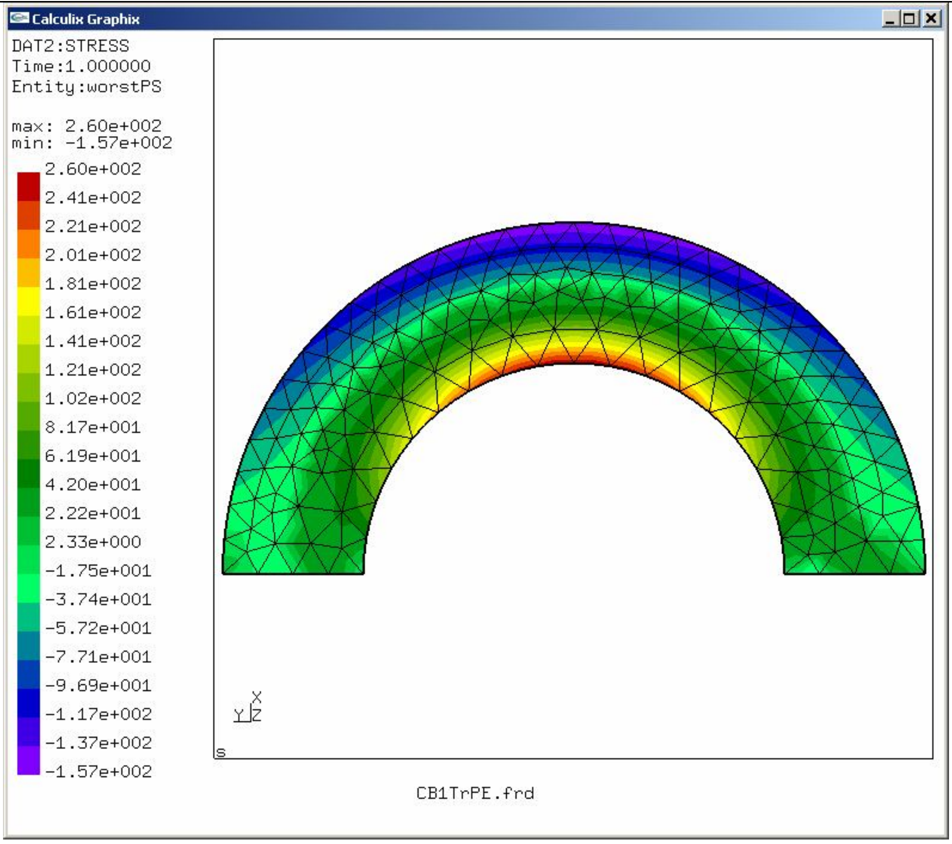


fig. 21: Worst principal stresses with CPE6 elements

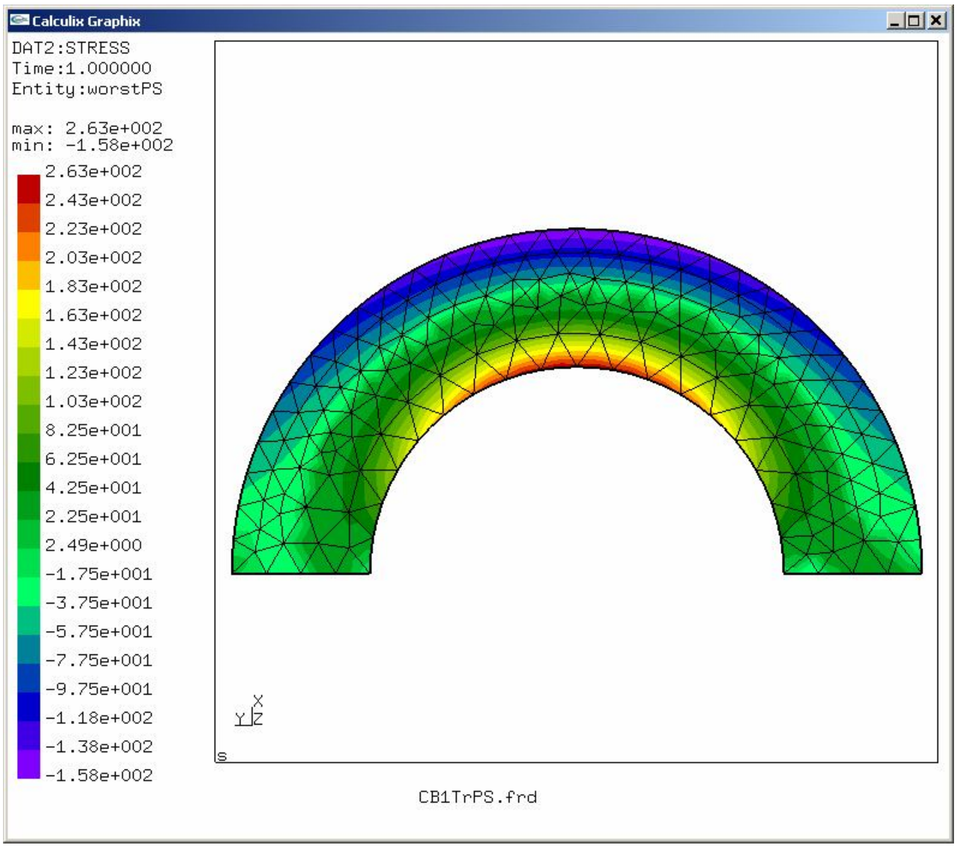


fig. 22: Worst principal stresses with CPS6 elements

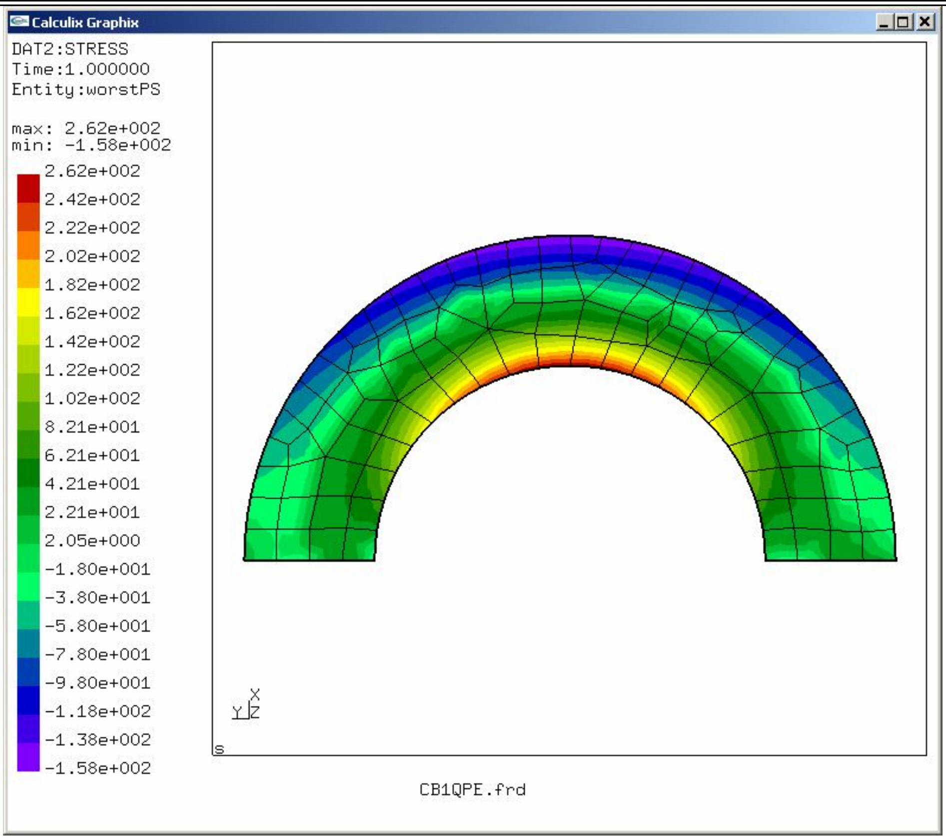


fig. 23: Worst principal stresses with CPE8 elements

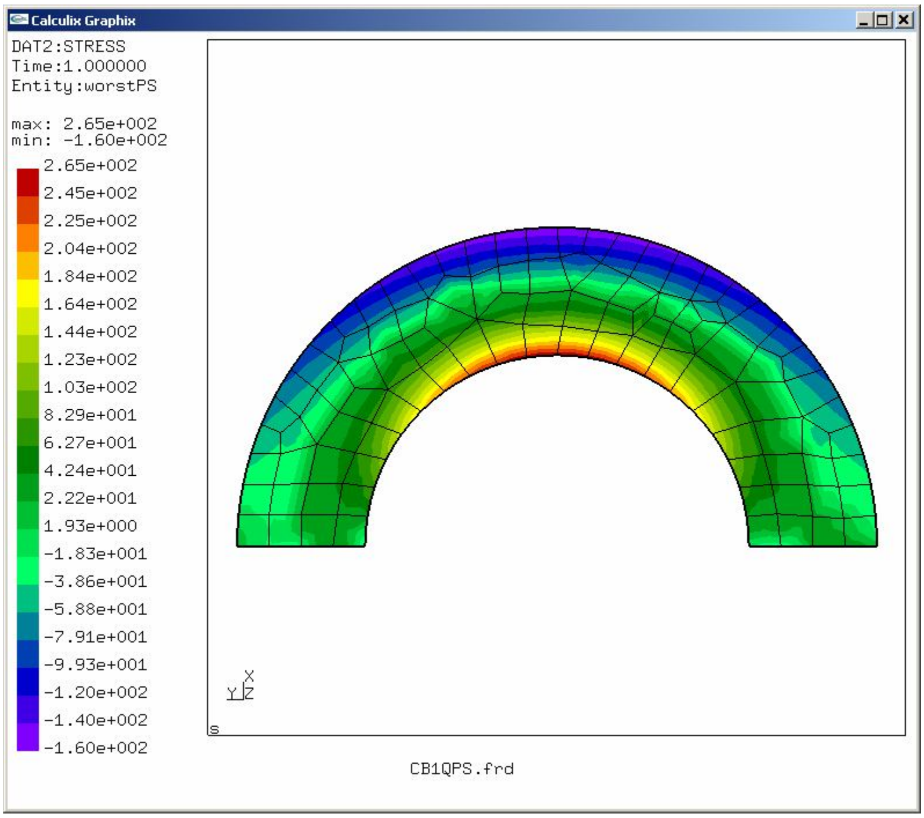


fig. 24: Worst principal stresses with CPS8 elements

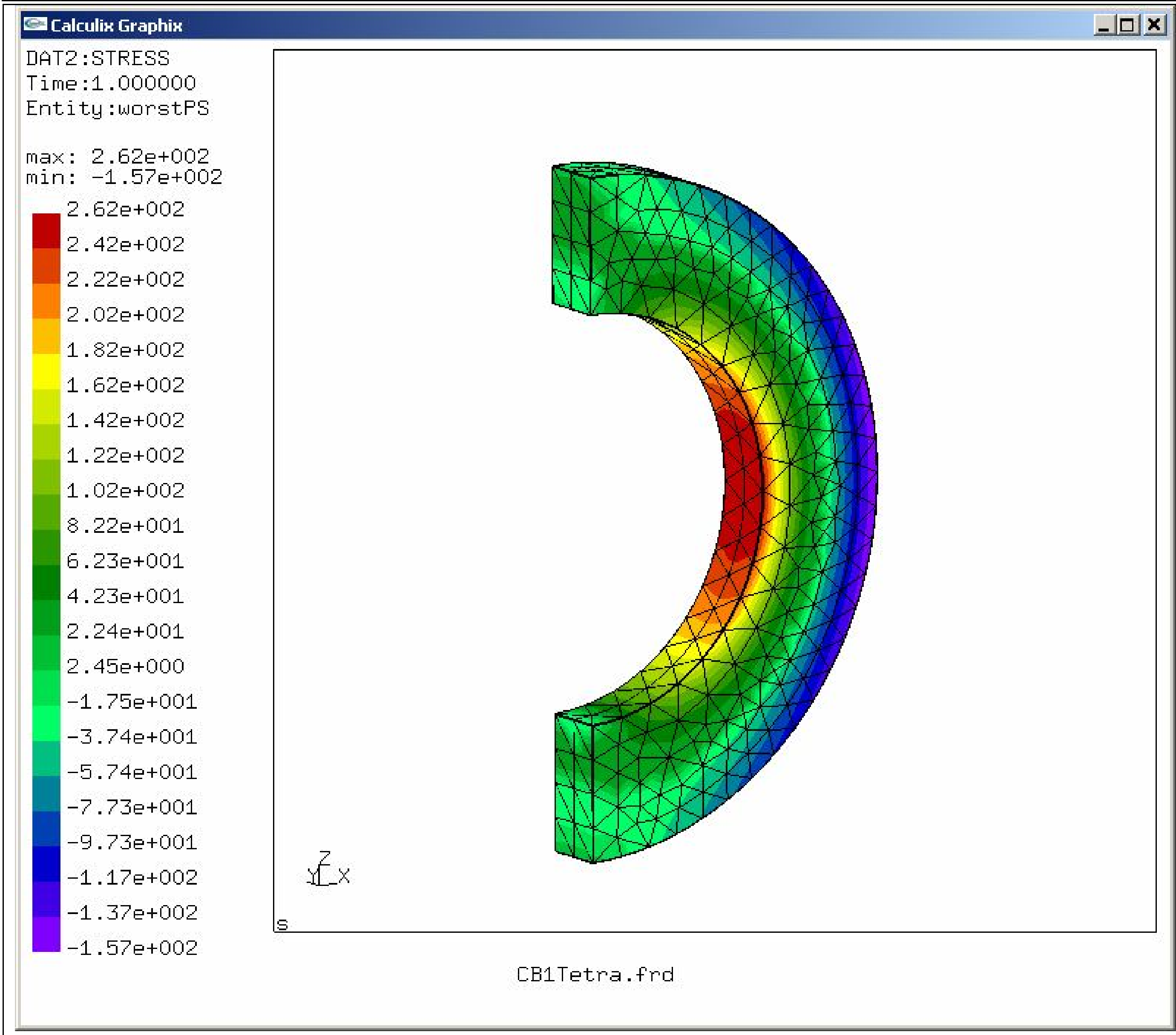


fig. 25: Worst principal stresses with C3D10 elements

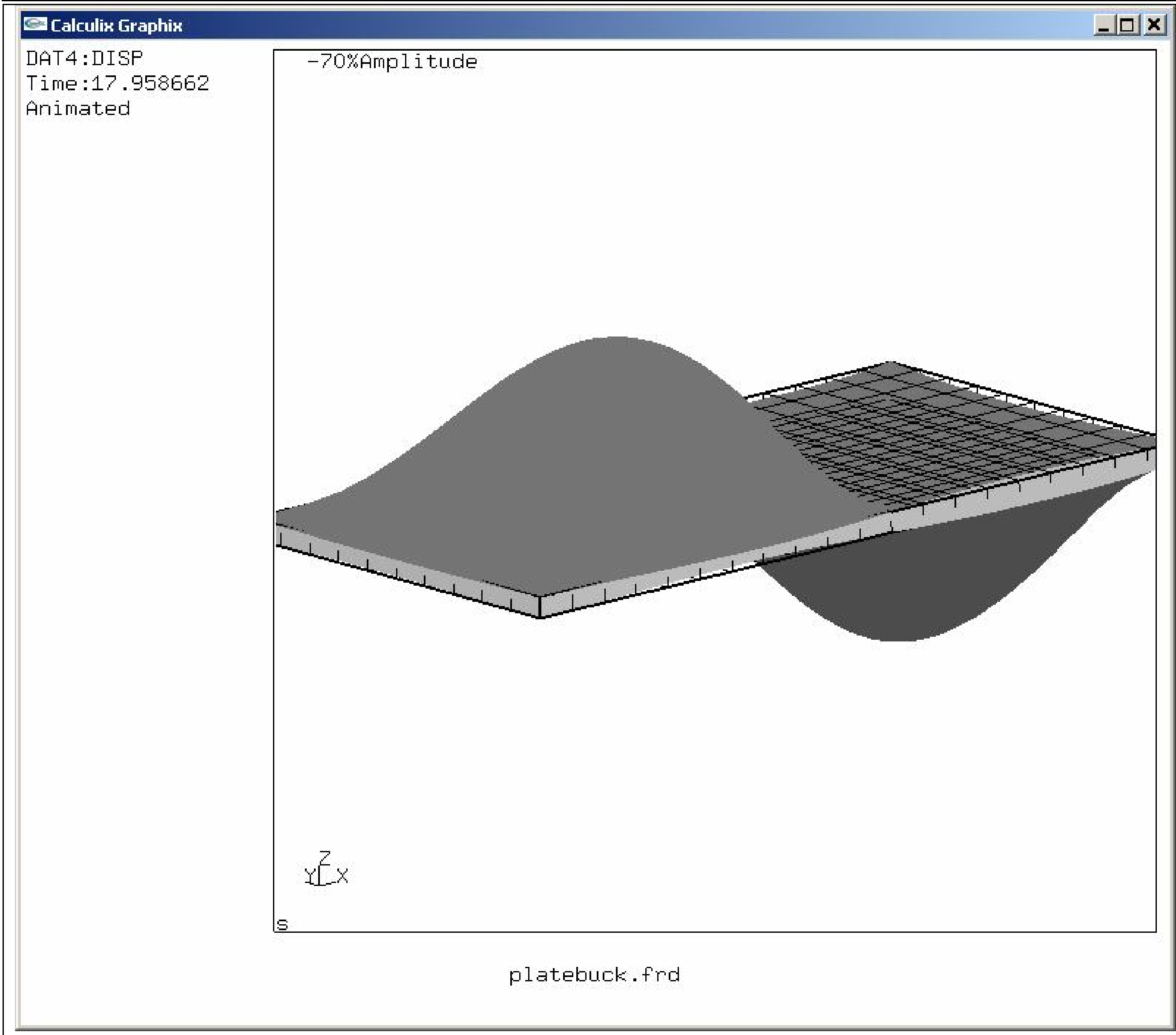
9. Buckling of a rectangular plate under compression

A rectangular plate (100x50x2,5 mm) simply supported at the edges, is subjected to a uniform compression, along the short edges, of 100 MPa.
$E = 200000$ MPa; $\nu = 0,3$

(File: Buck1S8.inp;)

Theory	CCX (S8)
Buckling factor	
18,077	17,959

Note: The buckling factor(s) is stored in dat file.



10. NAFEMS LE1

From NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990, the result for a elliptic membrane subjected to uniform outward pressure is σ_{yy} = 92,7 MPa at point D (see next figure).

Material: Linear elastic, Young’s modulus = 210 GPa, Poisson’s ratio = 0.3
Target solution: Tangential edge stress σ_{yy} at D is 92.7MPa

In CCX the results is obtained from several type of elements.

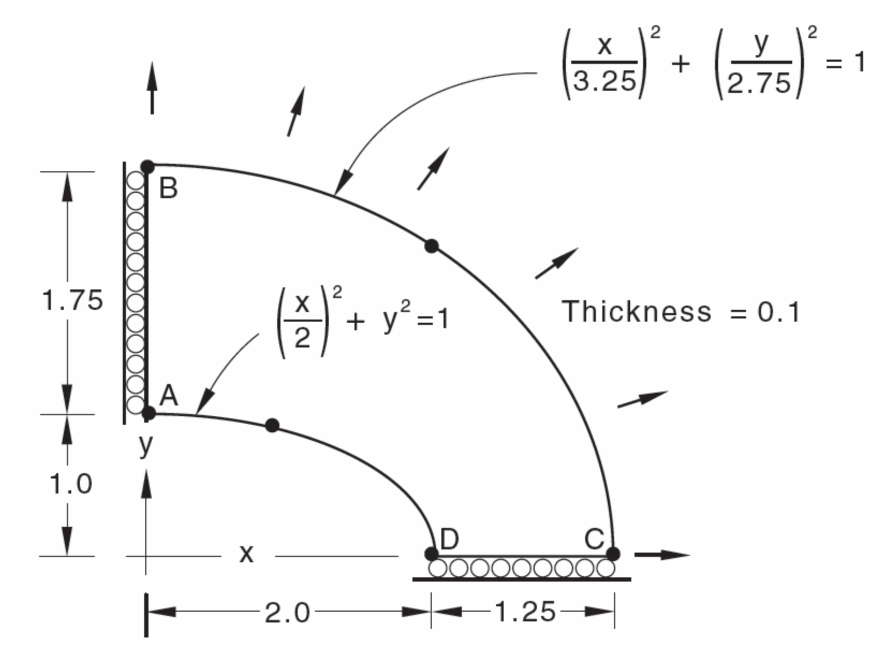


fig. 26: Elliptic membrane

NAFEMS	CCX (CPS8)	CCX (CPE8)	CCX (CPE6)	CCX (CPS6)	**CCX (CPE6)** Mesh refinement
Tangential stress at point B (MPa)					
92,7	92,8 +0,1%	93,3 +0,6%	88,6 -4,4%	88,5 -4,5%	92,5 -0,2%

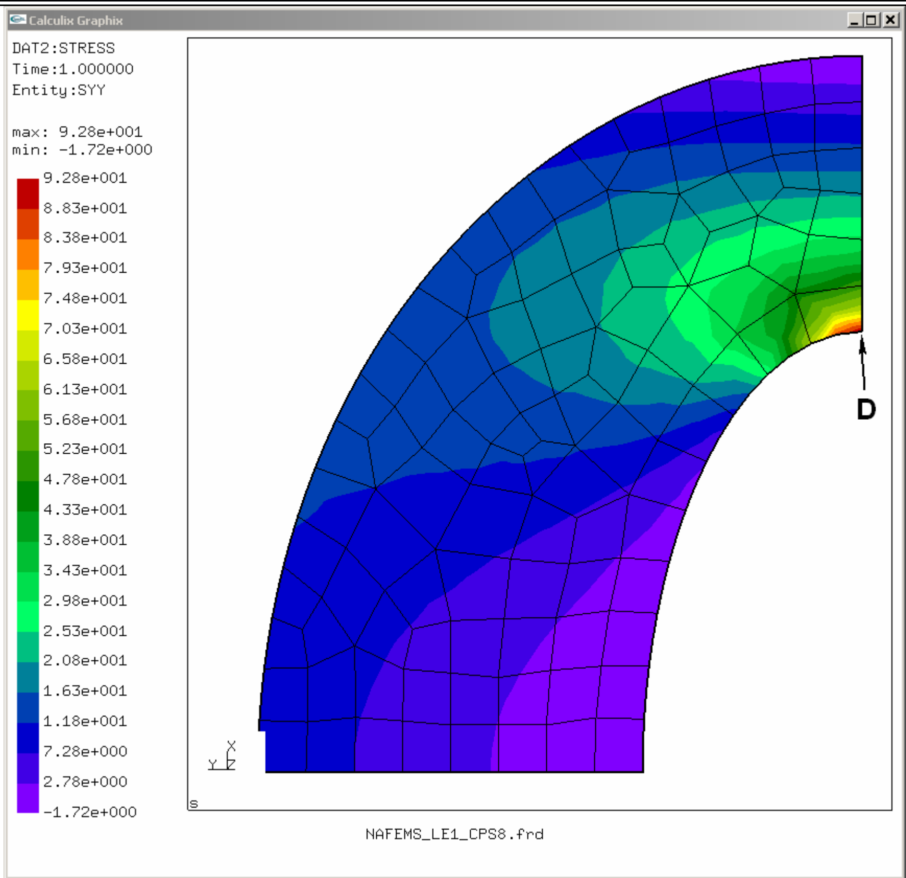


fig. 27: Tangential y-y stress with CPS8 elements

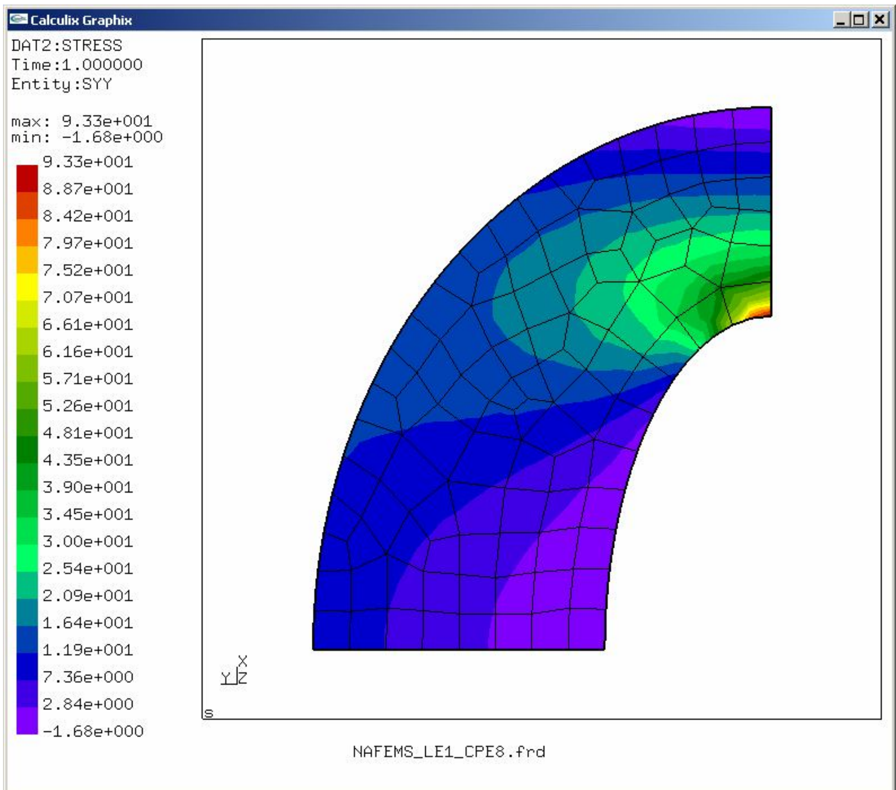


fig. 28: Tangential y-y stress with CPE8 elements

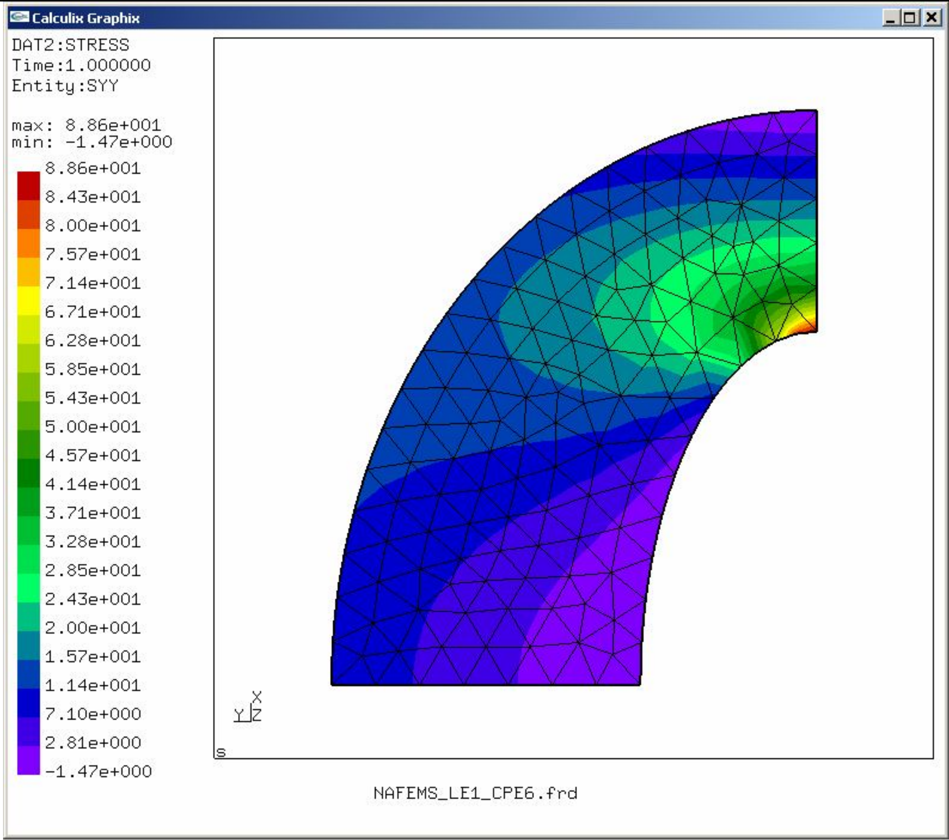


fig. 29: Tangential y-y stress with CPE6 elements

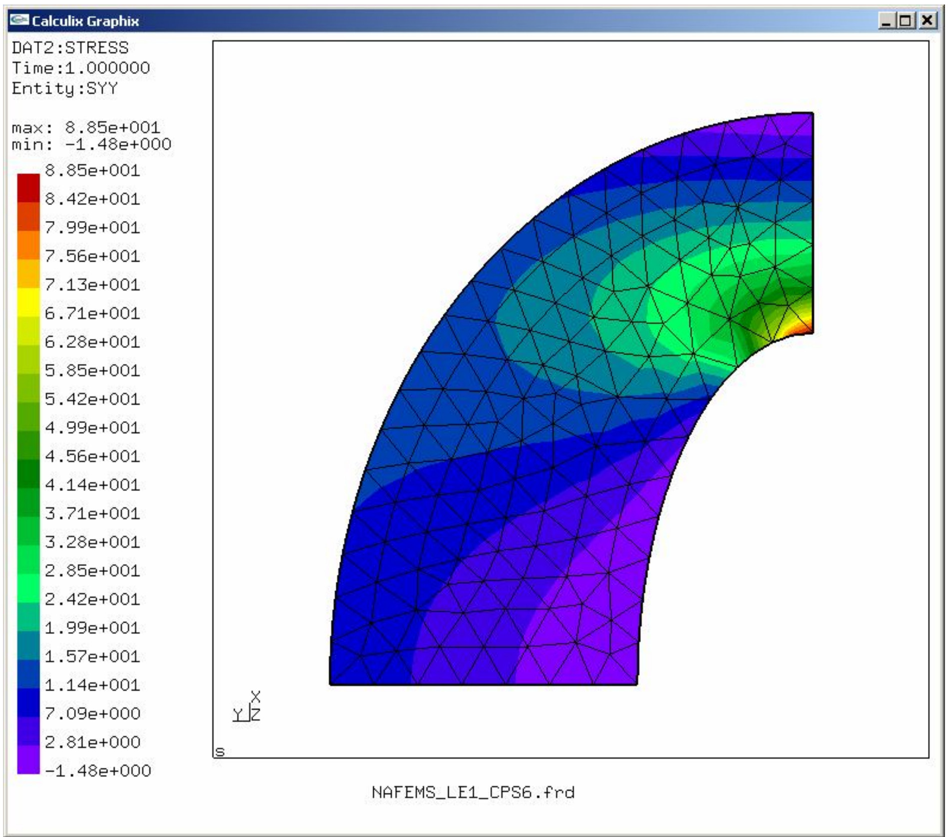


fig. 30: Tangential y-y stress with CPS6 elements

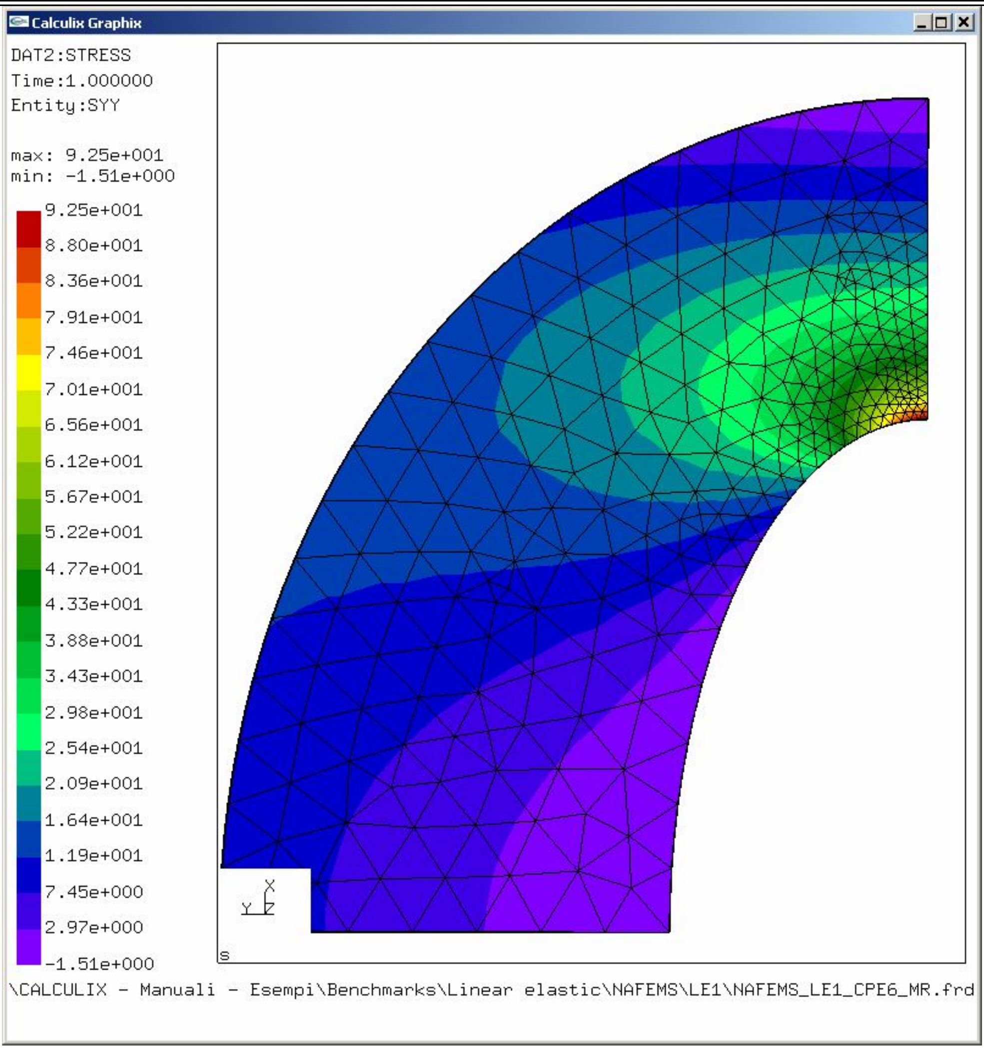


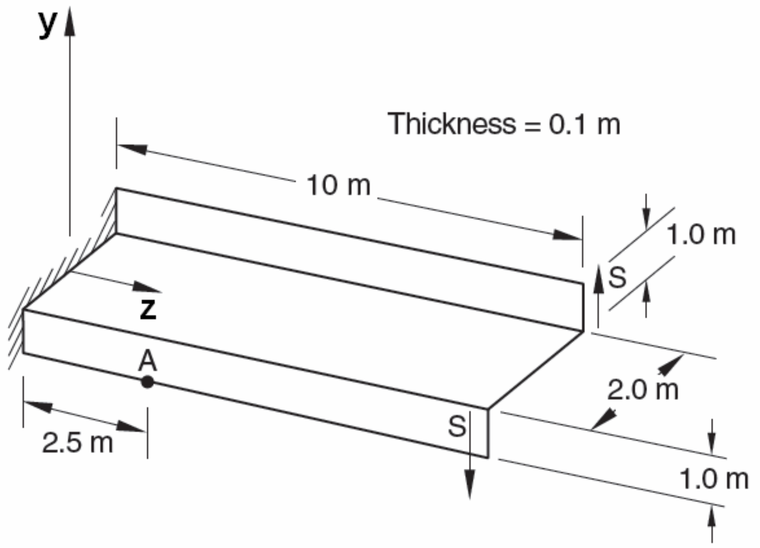
fig. 31: Tangential y-y stress with CPE6 elements and mesh refinement at a point

11. NAFEMS LE5

Test LE5 from NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.
Target solution: Axial stress, σ_{zz} = - 108 MPa at mid-surface, point A.

Material: Linear elastic, Young’s modulus = 210 GPa, Poisson’s ratio = 0.3

The cantilever is subjected to a torsion of 1,2 MNm



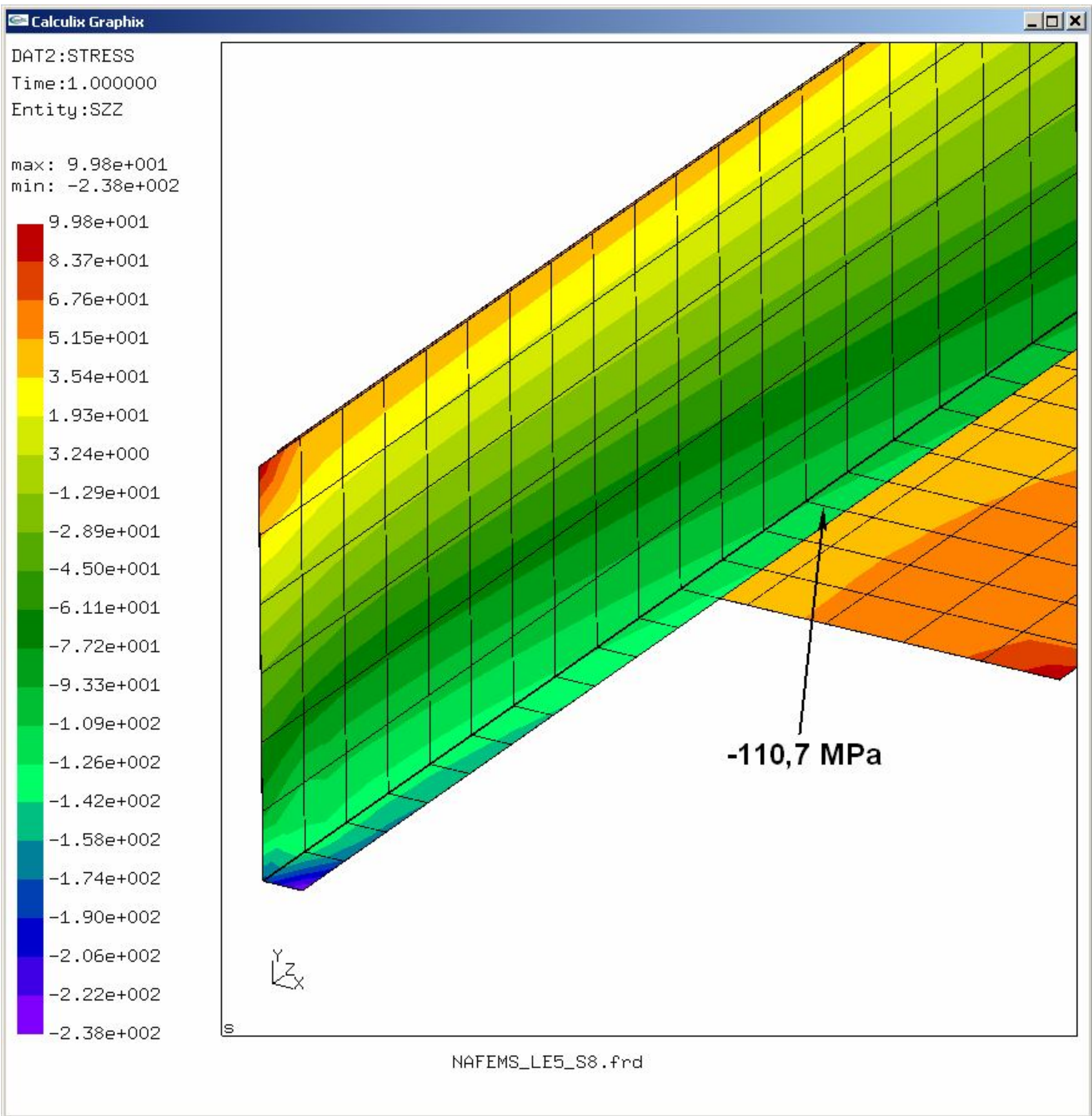
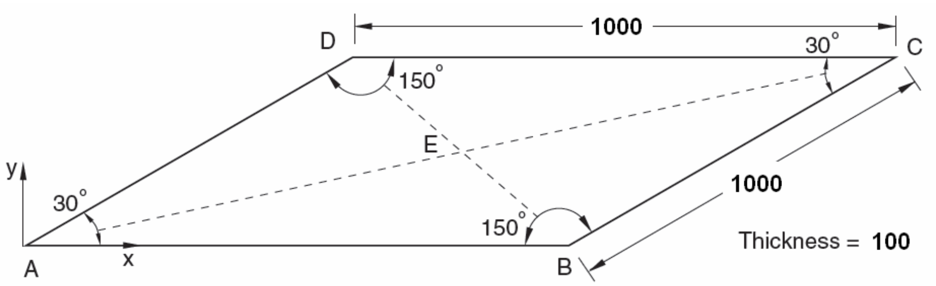


fig. 32: σ_{zz} stress at point A (Mid-surface)

12. NAFEMS LE6

Test LE6 from NAFEMS Publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.
Target solution: Maximum principal stress = 0.802 MPa on the lower surface at point E.

Material: Linear elastic, Young’s modulus = 210 GPa, Poisson’s ratio = 0.3
Plate subjected to uniform pressure of 0,7 KPa



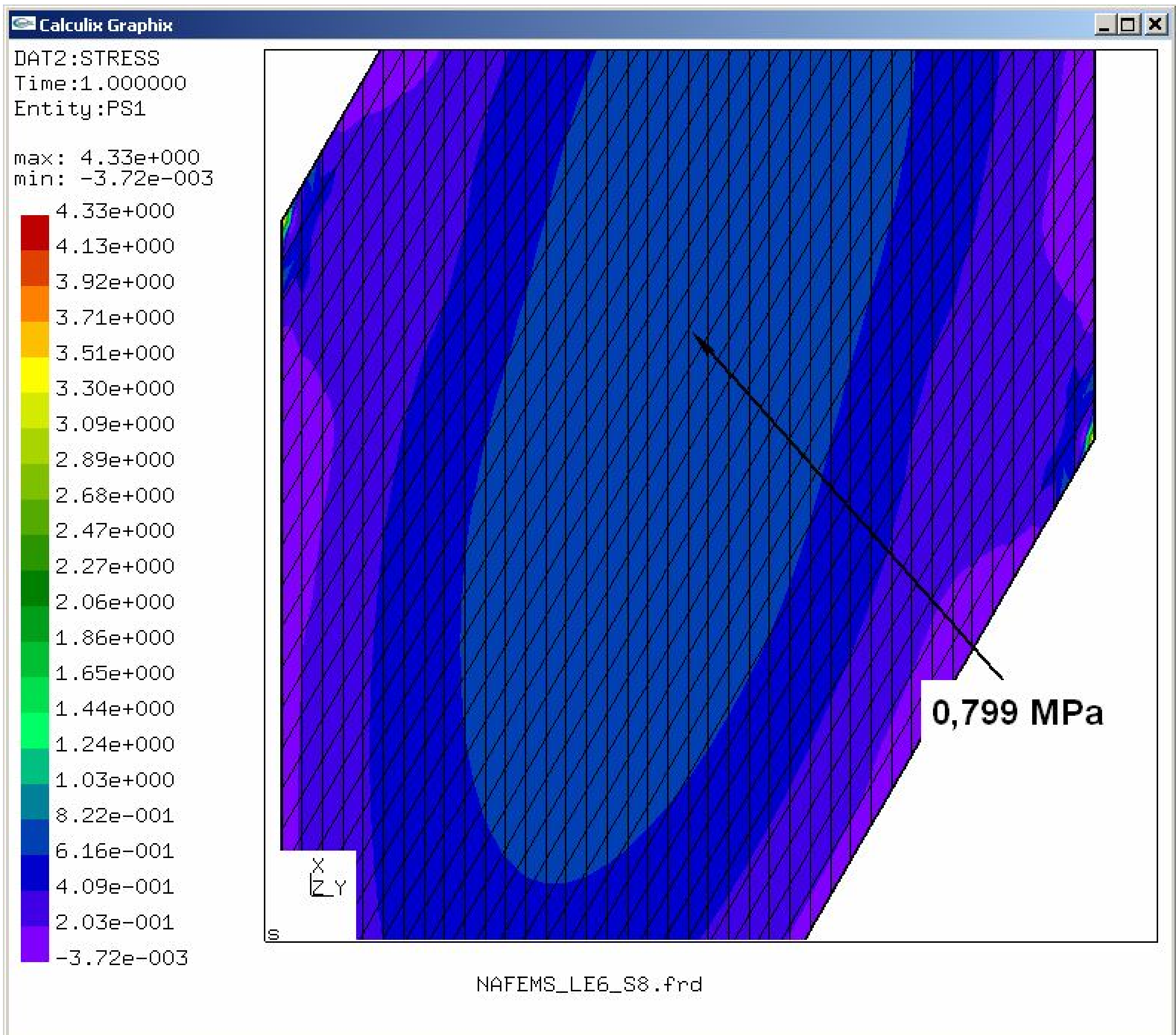


fig. 33: Maximum principal stress at point E (Bottom surface)

With coarse mesh the results are less accurate.

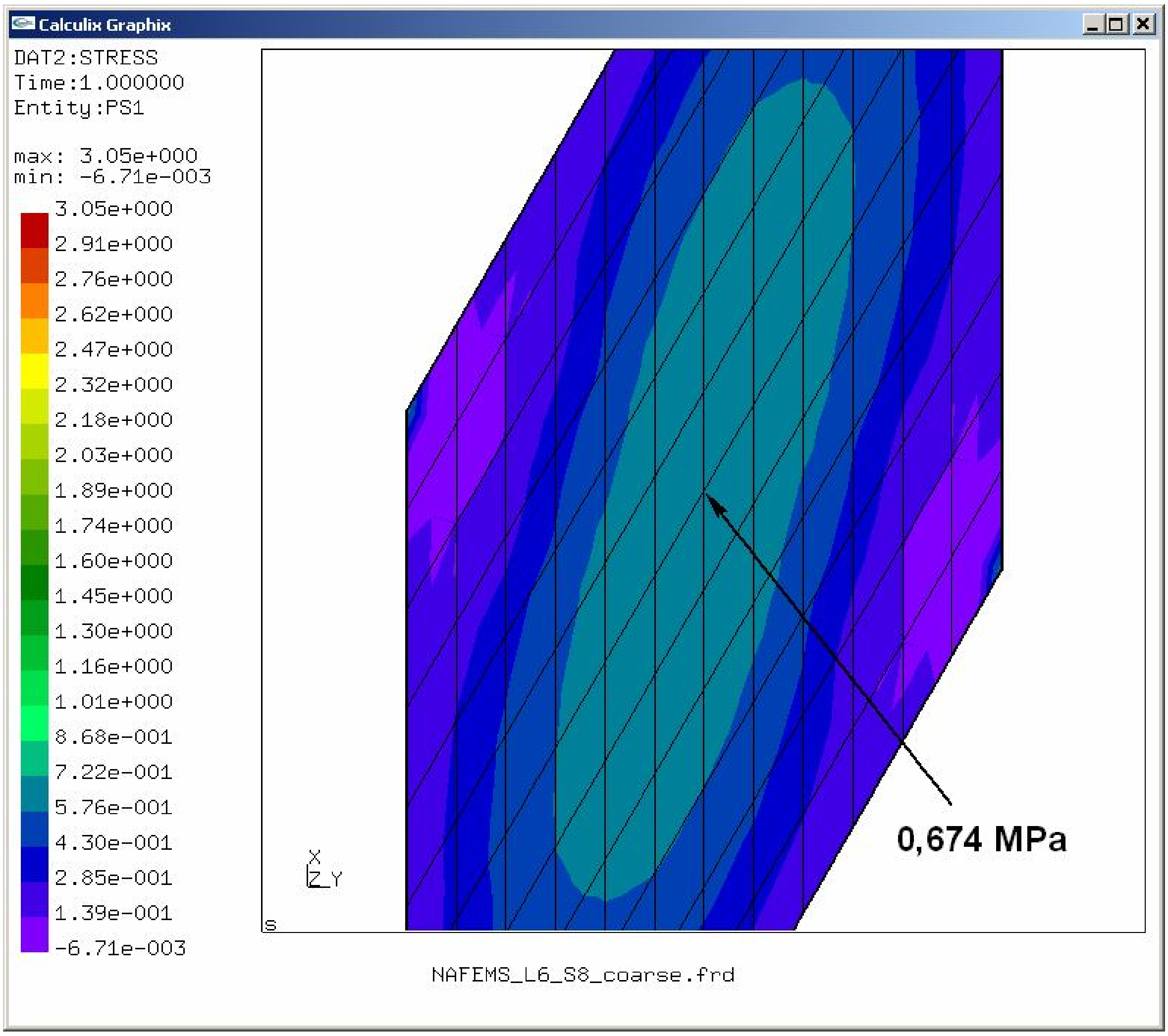


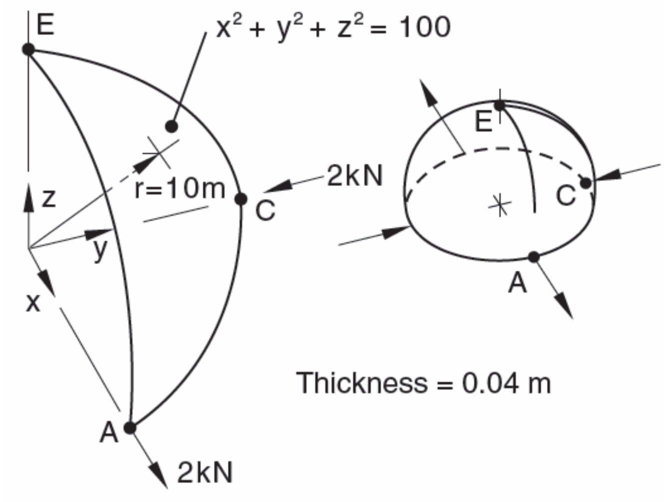
fig. 34: Maximum principal stress at point E (Bottom surface)

13. NAFEMS LE3

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE3 from NAFEMS publication TNSB, Rev. 3, "The Standard NAFEMS Benchmarks," October 1990.

Target solution: u_x = 185 mm at point A.

CCX results with S8 shell elements: u_x = 193 mm; error = +4,3%



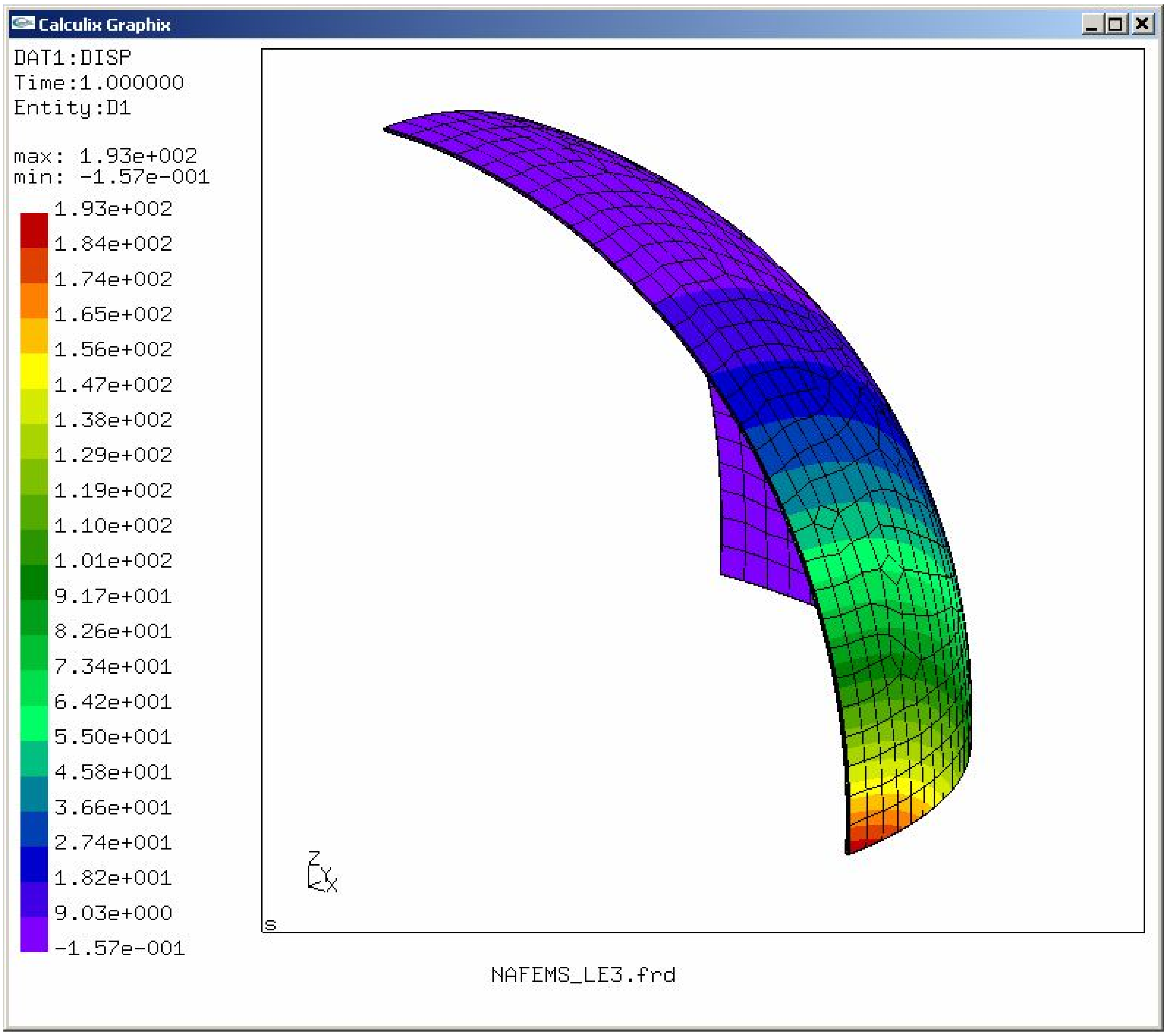


fig. 35: Displacement on x-direction

14. References

"Roark's formulas for stress and strain" – Seventh edition – McGraw Hill
"The Standard NAFEMS Benchmarks" October 1990