Coriolis Force Applied to Vibrating Mass Gyroscope (i.e. MEMS):

$v(t) = v_{max} \cdot sin(2 \cdot \pi \cdot f \cdot t)$

$r(t) = v(t) \cdot |t|$

$\theta(t) = -\omega \cdot |t|$

$s(t) = r(t) \cdot \theta(t)$

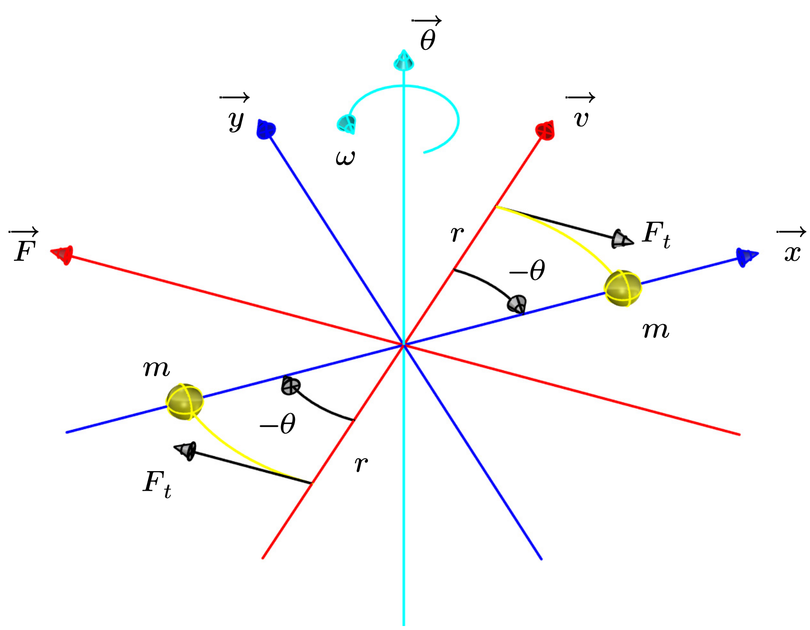
$v_t(t) = \frac{d}{dt} s(t) = \frac{d}{dt}\left(-\omega \cdot v \cdot t^2\right) = -2 \cdot t \cdot v \cdot \omega$

$a_t(t) = \frac{d}{dt} v_t(t) = \frac{d}{dt}\left(-2 \cdot t \cdot v \cdot \omega\right) = -2 \cdot v \cdot \omega$ Coriolis Acceleration

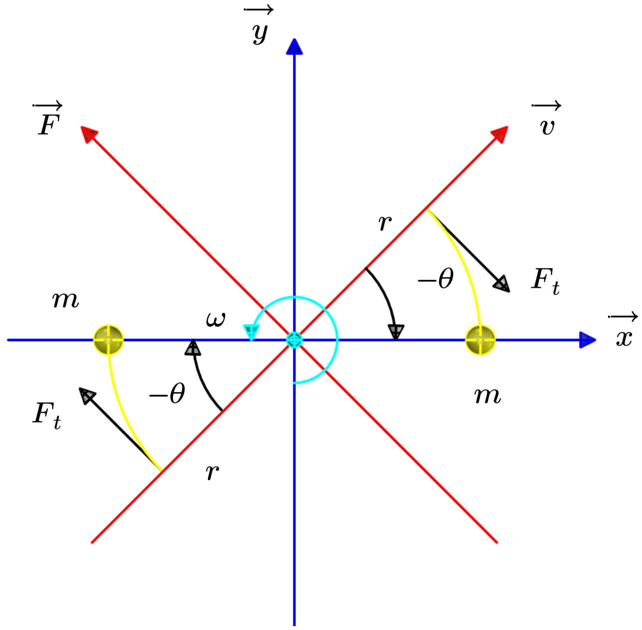
$F(t) = m \cdot a_t(t) = -2 \cdot m \cdot v \cdot \omega$ Coriolis Force

$x(t) = r(t) \cdot cos(\theta(t))$

$y(t) = r(t) \cdot sin(\theta(t))$



MEMS Sketch (3D View)



MEMS Sketch (2D View)

Coriolis Force Applied to Spinning Mass Gyroscope:

$\theta_1 = \omega \cdot t \qquad \theta_2 = \Omega \cdot t$

$\theta_3 = -\left(\theta_2 \times \theta_1\right) = \left(\left(\Omega \cdot t\right) \cdot \left(\omega \cdot t\right)\right) = -\Omega \cdot \omega \cdot t^2$ Angular Displacement about Output Axis

$\frac{d}{dt}\theta_3 = -2 \cdot \Omega \cdot \omega \cdot t$ Angular Velocity about Output Axis

$\alpha_3 = \frac{d^2}{dt^2}\theta_3 = -2 \cdot \Omega \cdot \omega$ Angular Acceleration about Output Axis

$s_3 = r \cdot \theta_3 = -r \cdot \Omega \cdot \omega \cdot t^2$ Arc Length Traveled about Output Axis

$v_t = \frac{d}{dt} s_3 = -2 \cdot r \cdot \Omega \cdot \omega \cdot t$ Tangential Velocity

$a_t = \frac{d}{dt} v_t = -2 \cdot r \cdot \Omega \cdot \omega$ Tangential Acceleration

$F_t = m \cdot a_t = -2 \cdot m \cdot r \cdot \Omega \cdot \omega$ Coriolis Force

The negative sign is due to the order the cross product was taken in. Reversing the order leads to positive values

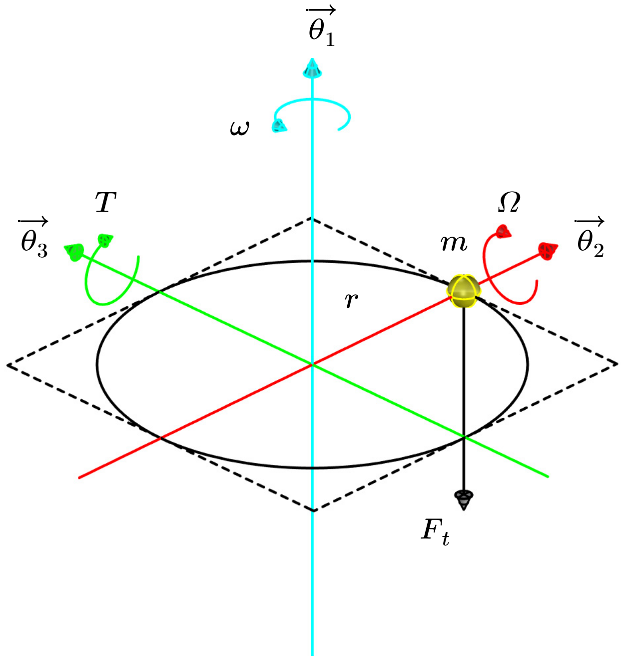
Note; $I_1 = m \cdot r^2 \qquad I_2 = 0 \qquad I_3 = \frac{1}{2} \cdot m \cdot r^2$

$H = I_1 \cdot \omega$ Angular Momentum about the Input Axis

$T_3 = H \cdot \Omega = I_1 \cdot \omega \cdot \Omega$ Conservation of Angular Momentum / Gyroscopic Precession

$T_3 = I_3 \cdot \alpha_3 = \left(\frac{1}{2} \cdot m \cdot r^2\right) \cdot \left(-2 \cdot \Omega \cdot \omega\right) = -I_1 \cdot \Omega \cdot \omega$ Traditional definition of torque

The sign difference is due to the order of the cross product. Result can be positive or negative, depending on the convention used.



Gyroscope Sketch (3D View)